
Proceedings of:

Precision Interferometric Metrology

2005 Summer Topical Meeting

July 20 to 22, 2005

***The Inn at Middletown
Middletown, Connecticut***

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Preface

This book comprises the proceedings of the 2005 Summer Topical Meeting. The contributions reflect the authors' opinions and are published as presented to ASPE, without change. Their inclusion in this publication does not necessarily constitute endorsement by the ASPE, or its editorial staff.

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2005 Summer Topical Meeting

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2005 Summer Topical Meeting

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12:00			
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3:00			
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5:00			
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7:00		OPEN FORUM	
9:00			

Foreword

Applications such as flat panel displays, photo-lithography, automotive systems and basic science are driving demand for lower uncertainty measurements in ever more challenging environments. This meeting will cover new developments in interferometric metrology – including new concepts and analyses, instrument development and application examples. In addition to paper sessions, there will be open discussion sessions and opportunities to make brief informal presentations on a variety of topics of interest. The talks will cover a broad range of technologies and highlight potential areas for future development.

The objective of the meeting is to bring together researchers and practitioners from industry, government, and academia to provide a forum for the exchange of ideas. In addition to paper and poster presentations, we will have a panel discussion on current industry needs and the perceived relationship between those needs and current research.

2005 Summer Topical Meeting Co-Chairmen:

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National Institute of Standards and Technology

Peter de Groot and Chris J. Evans

Zygo Corporation

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J. Schmit, VEECO Corporation

T. L. Schmitz, University of Florida

G. Wilkening, Physikalisch-Technische Bundesanstalt

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Technical Papers



A S P E

Metrology for Space-based Science Missions

Lay, O. P.; Dubovitsky, S.; Peters, R. D. (Jet Propulsion Laboratory)

The desire for higher angular resolution, greater sensitivity and dynamic range has driven the trend towards large collecting area and ultra-precise optics on space-based observing systems. No longer capable of being launched as monolithic units, these systems are being implemented as deployable structures or distributed over multiple spacecraft. Precise metrology is increasingly important for the stabilization of these large distributed systems, in which tolerances are often measured in picometers.

There are a number of challenges to be faced in developing a metrology system for such applications. In addition to the great precision that is often demanded, the systems must be guaranteed to operate without human intervention for mission durations of 5 to 10 years, after surviving the stress and vibration of launch and the radiation environment of space. Mass and power are extremely valuable commodities, and materials must be carefully chosen to avoid contamination. The failure of the metrology could compromise missions that may cost in excess of \$1 billion.

A wide range of current and future space science missions are dependent on metrology. Example applications include Synthetic Aperture Radar (SRTM), measurement of the Earth's gravity field (GRACE), measurement of stellar positions (GAIA, SIM-PlanetQuest), direct imaging of Earth-like planets around other stars (TPF-C, Darwin, TPF-I), detection of gravity waves (LISA), and imaging of black hole event horizons (Black Hole Imager). This presentation will describe two of these— the SIM-PlanetQuest and TPF-I missions — in more detail, followed by a summary of the MSTAR absolute metrology system that has been developed at the Jet Propulsion Laboratory.

The Space Interferometry Mission (SIM) is an optical interferometer with a baseline ~10 m, designed to measure the positions of stars to within 4 microarcseconds – an exquisite level of accuracy far beyond the current capability. Scheduled for launch in 2010 with a cost of \$1 billion, SIM is essentially one big metrology experiment, demanding precision at the 10's of picometer level. An extensive technology development effort has demonstrated the technical feasibility of the mission, and detailed design is underway. The basic principle of operation is described.

The Terrestrial Planet Finder Interferometer (TPF-I) will directly image Earth-like planets around nearby stars at mid-infrared wavelengths. To achieve the angular resolution needed to separate the planet from the star, the collecting apertures are located on separate spacecraft flying in formation, and relay their beams to a central combiner spacecraft. The array is phased to null the light from the star which is some 10 million times brighter than the planet. Effective nulling requires that the pathlengths through the instrument are controlled at the 1 nm level.

The MSTAR (Modulation Sideband Technology for Absolute Ranging) system was designed to address the needs of missions such as TPF-I. Based on phase modulation of a single stable laser, we have demonstrated absolute range accuracy of 100 nm in the lab, sufficient to resolve the integer cycle ambiguity of the optical phase measurement. The system is briefly described, along with a plan to demonstrate MSTAR on the Space Technology 9 mission.

Gravitational wave detection using precision interferometry

Gregory M. Harry, on behalf of the LIGO Scientific Collaboration
LIGO Laboratory, Massachusetts Institute of Technology,
NW17-161, Cambridge MA, 02139

Gravitational waves are a prediction of Einstein's general theory of relativity. Detection of these waves would further our understanding of gravity, however, the predicted amplitude is extremely small. A typical strain predicted from an astronomical source of gravitational waves measured on the Earth is about 10^{-21} . Detectors are currently operating around the world, including two locations in the US in Louisiana and Washington state, to look for these waves.

To detect such a tiny strain, precision interferometry with very long arms is used. The detectors consist primarily of Michelson interferometers. Every optic is supported by vibration isolation stacks to reduce disturbances from seismic noise. Each optic is suspended in a wire pendulum, for additional isolation from seismic noise and so the mirror is free to interact with the gravitational wave in the sensitive direction of the interferometer. The arms of each interferometer are 4 kilometers long to increase the motion of the free-hanging mirrors from the given wave's strain. Each arm has an additional mirror at the input, forming a Fabry-Perot cavity. This allows the laser power which measures the position of the mirrors to be higher, increasing the signal. The output port is held at a dark fringe, so a further mirror at the bright port is used to reflect light back into the interferometer, further increasing power. Limiting noise sources are seismic motion at low frequencies (below about 40 Hz), thermal vibrations of the suspensions (between 40 Hz and 200 Hz), and shot noise from the laser at high frequencies (above 200 Hz). Noise from other sources, including laser frequency and amplitude noise, scattered light, and electronics, are reduced to below these levels. The operating detectors in the US are currently within a factor of two in most bands of achieving these goals.

Even with such an ambitious detector, an actual detection of a gravitational wave may not be likely until second generation interferometers are built. Noise in all frequency bands is planned to be reduced leaving thermal vibrations of the mirrors themselves as the limiting noise source in the most sensitive band. Much of this thermal noise comes from mechanical loss in the optical coatings of the mirrors. Development of low mechanical loss mirrors, while preserving the low optical loss, low scatter, and high reflectivity necessary for the interferometry is an area of active research. Doping with titania of silica/tantala dielectric optical coatings has shown the most promise of satisfying all of these requirements. Annealing, different coating conditions, and additional materials and dopants are also being researched to improve the coatings.

Displacement Laser Interferometry with Sub-nanometer Uncertainty

Cosijns, S. J. A. G. (ASML Netherlands B.V.);
Haitjema, H. (Mitutoyo Research Center Europe B.V.); and
Schellekens, P. H. J. (Eindhoven University of Technology)

When considering interferometric displacement measurements with nanometer uncertainty over small distances (below 1mm) the measurements are influenced by periodic deviations originating from polarization mixing. In measurements with nanometer uncertainty over larger distances this error may become negligible compared to errors introduced by the refractive index changes of the medium in which the measurement takes place.

A model based on Jones matrices enables the prediction of periodic deviations originating from errors in optical alignment and polarization errors of the components of the interferometer. In order to enable the incorporation of polarization properties of components used in interferometers, different measurement setups are discussed. Measurement setups are used to measure the polarization properties of a heterodyne laser head used in the interferometer system. Based on ellipsometry a setup is realized to measure the polarization properties of the optical components of the laser interferometer.

With use of measurements carried out with these setups and the model it can be concluded that periodic deviations originating from different error sources can not be superimposed, as interaction exists which may cause partial compensation.

The correctness of the predicted periodic deviations was examined by placing an entire interferometer system on a traceable calibration setup based on a Fabry-Pérot interferometer. This system enables a calibration with an uncertainty of 0.94nm over a range of 300 μ m. Prior to this measurement the polarization properties of the separate components were measured to enable a good prediction of periodic deviations with the model. The measurements compared to the model revealed a standard deviation of 0.14nm for small periodic deviations and a standard deviation of 0.3nm for periodic deviations with amplitudes of several nanometers.

This Jones model combined with the setups for measurement of the polarization properties forms a practical tool which enables the designer to choose the right components and alignment tolerances for a practical setup with (sub-)nanometer uncertainty specifications.

To improve the uncertainty of existing laser interferometer systems a compensation method for heterodyne laser interferometers was investigated. It is based on phase quadrature measurement in combination with a compensation algorithm based on

Heydemann's compensation which is used frequently in homodyne interferometry. The system enables a compensation of periodic deviations with an amplitude of 8nm down to an uncertainty of 0.2nm. From measurements it appears that ghost reflections occurring in the optical system of the interferometer cannot be compensated by this method.

Regarding the refractive index of air three measurement methods were compared. The three empirical equations which can be found in literature, an absolute refractometer based on a commercial interferometer and a newly developed tracker system based on a Fabry-Pérot cavity. The tracker was tested to investigate the feasibility of the method for absolute refractometry with improved uncertainty. The developed tracker had a relative uncertainty of $8 \cdot 10^{-10}$. The comparison revealed some temperature effects which cannot be explained yet. However the results of the comparison indicate that an absolute refractometer based on a Fabry-Pérot cavity will improve the uncertainty of refractive index measurement compared to existing methods.

Real-time periodic error correction: experiment and data analysis

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Keywords: Interferometry; heterodyne; displacement; nonlinearity; cyclic

Abstract

This paper provides experimental validation of the digital first-order periodic error reduction scheme described by Chu and Ray. A bench-top setup of a single-pass, heterodyne Michelson interferometer, designed to minimize common error contributors such as Abbe, dead path, and environment, is described. Linear, reciprocating motion generation is achieved using a parallelogram, leaf-type flexure. Periodic error amplitude is varied through independent rotation of a half wave plate and polarizer. Experimental results demonstrate that the correction algorithm can successfully attenuate first-order error to sub-nm levels for a wide range of frequency mixing conditions.

1. Introduction

Differential-path interferometry is used extensively in situations requiring accurate displacement measurements. Examples include lithographic stages for semiconductor fabrication, transducer calibration, and axis position feedback for precision cutting and measuring machines. In many applications, a dual frequency (heterodyne) Michelson-type interferometer with single, double, or multiple passes of the optical paths is implemented. These systems infer changes in displacement of a selected optical path by monitoring the optically-induced variation in a photodetector current. The phase-measuring electronics convert this photodetector current to displacement by digitizing the phase progression of the photodetector signal. Due to non-ideal performance, mixing between the two heterodyne frequencies may occur, which results in periodic errors superimposed on the desired displacement data (i.e., the error amplitude varies cyclically with the target position). In practice, first-order periodic error, which appears as single sideband modulation on the data at a spatial frequency of one cycle per displacement fringe, often dominates. Second-order periodic error, with a spatial frequency of

two cycles per displacement fringe, is also commonly observed.

Although modifications to traditional optical setups may be implemented to reduce periodic error, it is often inconvenient to make changes to existing configurations. As an alternative to these changes, Chu and Ray have recently described a scheme to correct first-order periodic error in real time using digital logic hardware [1]. The approach is based on a modification of the classic pseudo-inverse linear regression solution using a quantized operator, so that digital accumulators may be applied to fit and correct the error in real time on a continuous basis.

The purpose of this study is to validate of the Chu and Ray approach using a bench-top setup of a single-pass, heterodyne Michelson-type interferometer. The setup enables: 1) isolation of periodic error as the primary uncertainty source in displacement measuring interferometry; and 2) variation of the frequency mixing that leads to periodic error so that the error amplitude may be changed. During target motion, the real time first-order error correction is digitally applied in hardware and both the corrected and uncorrected measurement signals are recorded. Various frequency mixing levels are realized by adjustment of the setup optics; the periodic error levels before and after correction are presented for multiple cases.

2. Background

In this work we focus on heterodyne Michelson-type interferometers. In these systems, imperfect separation of the two light frequencies into the measurement (moving) and reference (fixed) paths has been shown to produce first- and second-order periodic errors. The two heterodyne frequencies are typically carried on collinear, mutually orthogonal, linearly polarized laser beams in a method referred to as polarization-coding. Unwanted leakage of the reference frequency into the measurement path, and vice versa, may occur due to non-orthogonality between the ideally linear beam polarizations, elliptical polarization of the

individual beams, imperfect optical components, parasitic reflections from individual optical surfaces, and/or mechanical misalignment between the interferometer elements (laser, polarizing optics, and targets). In a perfect system, a single wavelength would travel to a fixed target, while a second, single wavelength traveled to a moving target. Interference of the combined signals would yield a perfectly sinusoidal trace with phase that varied, relative to a reference phase signal, in response to motion of the moving target. However, the inherent frequency leakage in actual implementations produces an interference signal which is not purely sinusoidal (i.e., contains spurious spectral content) and leads to periodic error in the measured displacement.

Fedotova [2], Quenelle [3], and Sutton [4] performed early investigations of periodic error in heterodyne Michelson interferometers. Subsequent publications identified and described these periodic errors and built on the previous work [5-32]. Specific areas of research have included efforts to measure periodic error under various conditions [e.g., 5-8], frequency domain analyses [9-11], analytical modeling techniques [12-16], Jones calculus modeling methods [8,17], and reduction of periodic errors [e.g., 9,18,30,32]. Schmitz and Beckwith [33] summarize the potential periodic error contributors using a Frequency-Path, or F-P, model, which identifies each possible path for each light frequency from the source to detector and predicts the number of interference terms that may be expected at the detector output.

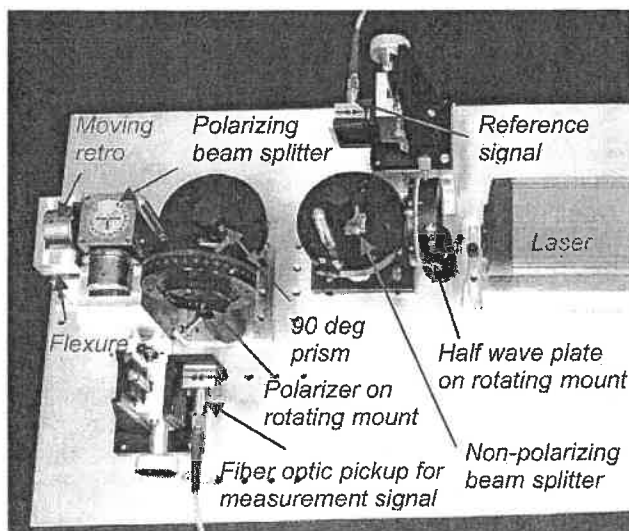


Figure 1: Photograph of bench-top setup for single-pass heterodyne interferometer.

3. Experimental setup description

A photograph of the setup is provided in Fig. 1. The orthogonal, linearly polarized beams with a split frequency of approximately 3.65 MHz generated within the Helium-Neon laser first pass through a half wave plate. Rotation of the half wave plate enables variation in the apparent angular alignment (about the beam axis) between the polarization axes and polarizing beam splitter; deviations in this alignment lead to frequency mixing in the interferometer. The light is then incident on a non-polarizing beam splitter (80% transmission) that directs a portion of the beam to a fiber optic pickup after passing through a fixed angle sheet polarizer (oriented at nominally 45 deg to the laser orthogonal polarizations). The pickup is mounted on a two rotational degree-of-freedom flexure which enables efficient coupling of the light into the multi-mode fiber optic. This signal is used as the phase reference in the measurement electronics.

The remainder of the light continues to the polarizing beam splitter where it is nominally separated into its two frequency components that travel separately to the moving and fixed retroreflectors. In this design, motion of the moving retroreflector is achieved using a parallelogram, leaf-type flexure. This enables nominally linear, oscillating displacement at a single frequency. The harmonic motion profile: 1) includes constantly varying velocity, acceleration, and jerk levels that are conveniently adjusted by varying the displacement amplitude; and 2) provides a rigorous test of the digital periodic error correction algorithm which assumes negligible jerk. After the beams are recombined in the polarizing beam splitter, they are directed by a 90 deg prism through a polarizer. Rotation of the polarizer changes the relative amplitude of the intended and mixing-induced interference signals and, therefore, the periodic error. Finally, the light is launched into a fiber optic pickup. This serves as the measurement signal in the measurement electronics.

As noted, the intent of the setup design was to minimize other well-known error contributors [19,20,34] and enable variation in the periodic error nature (i.e., first- or second-order) and amplitude. To isolate periodic error, the setup was designed with zero Abbe offset (i.e., the measurement axis was collinear with the motion axis) and zero dead path difference (i.e., the

distance between the polarization beam splitter and the moving retroreflector was equal to the distance between the polarization beam splitter and the fixed retroreflector at initialization). The measurement time (100 ms) and motion amplitude ($<200\text{ }\mu\text{m}$) were kept small to minimize the contribution of air refractive index variations due to the environmental changes. Additionally, the small target motion amplitude resulted in small beam shear (using the flexure dimensions and material properties, the maximum parasitic displacement perpendicular to the beam axis was calculated to be 0.1 nm for a $100\text{ }\mu\text{m}$ motion amplitude) and angular error ($3\times 10^{-3}\text{ }\mu\text{rad}$ for a $100\text{ }\mu\text{m}$ motion amplitude) [35].

Error contributors which were not well-controlled by this setup include cosine error, or an angular misalignment between the measurement and motion axes, due to the small displacement range and mechanical noise (the flexure's low stiffness and light damping caused table vibrations to be transmitted to the moving retroreflector, although these were reduced somewhat by mounting the entire assembly on a rubber mat). However, this study is unique in that the error correction was applied digitally. The analog measurement signal was sampled (0.3 nm resolution) and then the first-order periodic error correction was applied to the digitized data. Because our intent was to compare the (digitized) corrected and uncorrected signals, the cosine and mechanical noise errors can be considered common mode and have little influence on the final results presented here.

4. Experimental results

In this section, we describe the analysis procedure used to extract periodic error from the moving retroreflector displacement and present results for various angular orientations of the half wave plate and polarizer. Data was collected by first initiating flexure motion using a light impact (applied by a rubber-tipped mallet) and then recording displacement during the resulting harmonic motion (312.5 kHz sampling frequency).

4.1 Data analysis method

For a unidirectional, constant velocity motion, periodic error can be identified in an interferometer signal by subtracting the least squares straight line fit from the data. In our case, the gross flexure motion is best

represented by an exponentially decaying sine wave with some initial phase and, potentially, a DC offset depending on the initial displacement value of the interferometer. To remove the gross motion and isolate the periodic error, a nonlinear least squares fit to the data was performed using a function of the form $x(t) = \delta + Ae^{-\zeta\omega_n t} \sin(\omega_d t + \alpha)$, where δ is the DC offset, A is the amplitude, ζ is the viscous damping ratio, ω_n is the undamped natural frequency, $\omega_d = \omega_n \sqrt{1 - \zeta^2}$ is the damped natural frequency, and α is the initial phase. Once the fit parameters were determined, this function was subtracted from the uncorrected and corrected (first-order error removed) signals and the periodic error levels compared. Typical values for ω_n and ζ were 248.6 rad/s (39.6 Hz) and 0.009 (0.9% damping), respectively.

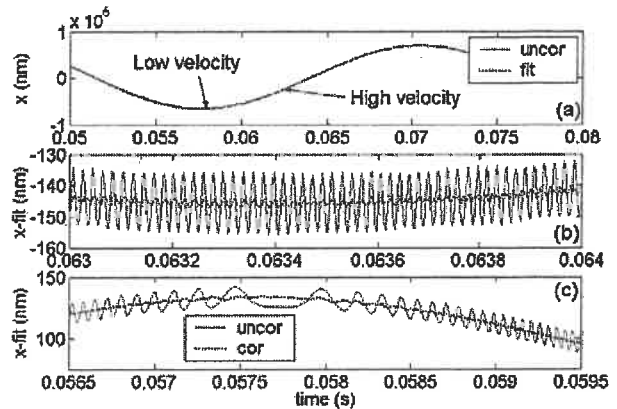


Figure 2: Example results from data analysis: a) uncorrected gross flexure motion with sinusoidal nonlinear least squares fit superimposed; b) difference between motion and fit in high velocity region; and c) difference in low velocity region.

Example results for the fitting procedure are provided in Fig. 2. A portion of the gross uncorrected motion (solid line) and nonlinear least squares fit (dotted line) are shown in panel a). These signals are then differenced to isolate the periodic error. Panel b) shows the result for a high velocity section of the original signal, while panel c) shows the result for a low velocity portion. In both cases, the dominant first-order error (the amplitude was set by angular misalignment of the polarizer) is effectively removed by the correction algorithm. It may also be noted that the least squares fit does not exactly capture the actual motion due to the small curvature and DC offset observed in both panels b) and c); however, the low spatial

frequency and DC offset errors do not significantly affect the subsequent frequency domain analysis of the periodic error amplitudes before and after correction.

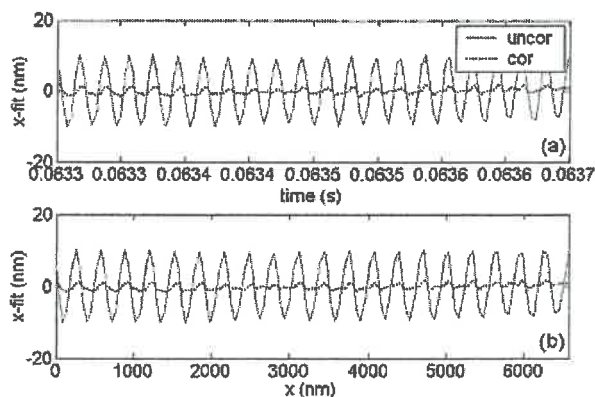


Figure 3: First-order periodic error reduction example: a) error vs. time for uncorrected (solid line) and corrected (dotted line) signals; b) error vs. uncorrected displacement.

A subset of the high velocity data shown in Fig. 2b) is reproduced in Fig. 3a) (the DC offset has been removed) and replotted versus uncorrected position in 3b). For the single pass Helium-Neon interferometer setup used here, first-order error repeats every $633/2 = 316.5$ nm, while second-order completes a full cycle in $633/4 = 158.3$ nm. It is now clearly observed that the signal is dominated by first-order error in this case. To identify the first- and second-order error amplitudes, the Fast Fourier transform of the error (vs. displacement) was computed and the spatial frequency axis normalized to periodic error order; see Fig. 4. It is shown that the first-order error magnitude has been reduced from 8.4 nm to 0.9 nm.

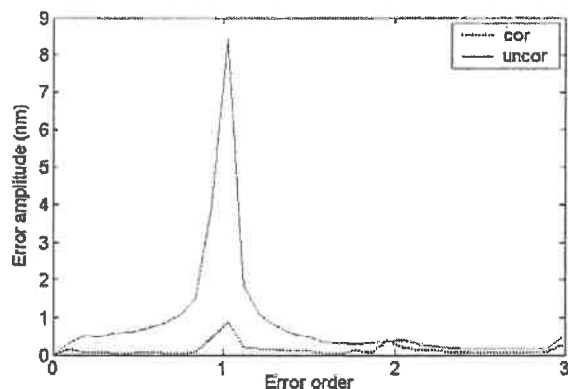


Figure 4: Fast Fourier transform of error vs. displacement data for corrected (dotted line) and uncorrected (solid line) signals. The first-order error is reduced from 8.4 nm to 0.9 nm.

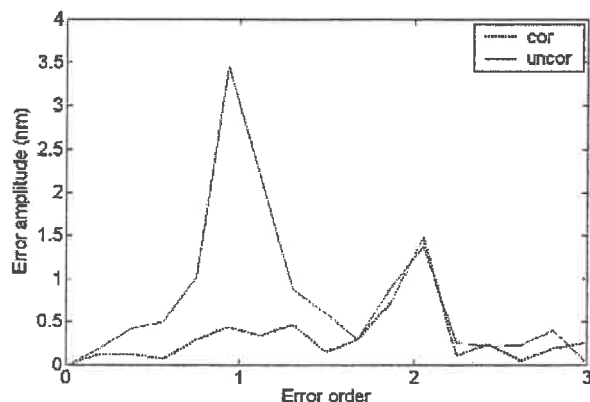


Figure 5: Fast Fourier transform for example with first- and second-order error. The first-order error is reduced from 3.5 nm to 0.4 nm.

A second example is shown in Fig. 5. In this case, both first- and second-order periodic error are present (due to angular misalignments of both the polarizer and half wave plate). It is seen that the first-order error is reduced from 3.5 nm to 0.4 nm, while the second-order error remains unaffected (the algorithm does not currently correct this error).

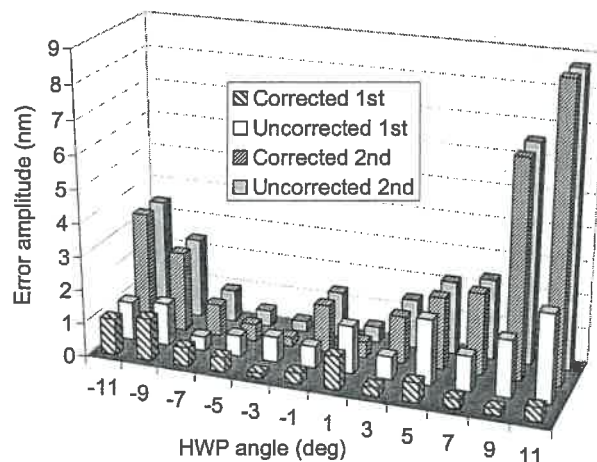


Figure 6: Variation in first- and second-order periodic error with changes in half wave plate orientation. Results for both the uncorrected and corrected signals are provided.

4.2 Error variation with interferometer setup

To explore the independent influences of the half wave plate and polarizer orientations, tests were carried out where one orientation was fixed and the other varied about a nominal value. To represent the results, the uncorrected and corrected first- and second-order periodic error amplitudes (using the Fourier transform data representations) were extracted for each orientation. Figure 6 shows the results for a fixed

(nominally 45 deg) polarizer angle and an ± 11 deg variation in the half wave plate angle. While the trend in both error orders is increased amplitude with larger departure from the nominal orientation, the second-order error is more strongly affected.

Figure 7 displays the results for a variable polarizer angle with a fixed half wave plate angle (the fast axis was nominally aligned with one of the linear polarization directions). In this case, the first-order error is more strongly influenced. In all instances, the correction scheme reduces the first-order periodic error to sub-nm levels. As before, the second-order amplitudes are not affected.

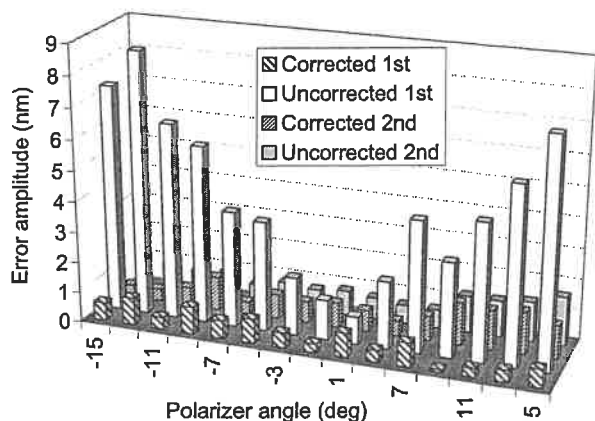


Figure 7: Variation in first- and second-order periodic error with changes in polarizer orientation. Results for both the uncorrected and corrected signals are provided.

4.3 Repeatability

The final measurement activity was to evaluate the repeatability of the periodic error amplitudes from test to test. To complete this task, displacement was recorded for 15 separate impacts of the flexure-mounted moving retroreflector. Half wave plate and polarizer angles were selected to minimize periodic error using the previous results. The analysis described in Section 4.1 was then completed for each data record and the first- and second-order periodic error amplitudes determined for both the uncorrected and corrected cases. The results are shown in Fig. 8. The standard deviations for the error amplitudes were less than 0.2 nm (first-order) and 0.3 nm (second-order) for the 15 data sets. This compares favorably with the 0.3 nm displacement resolution for the interferometer used in this study.

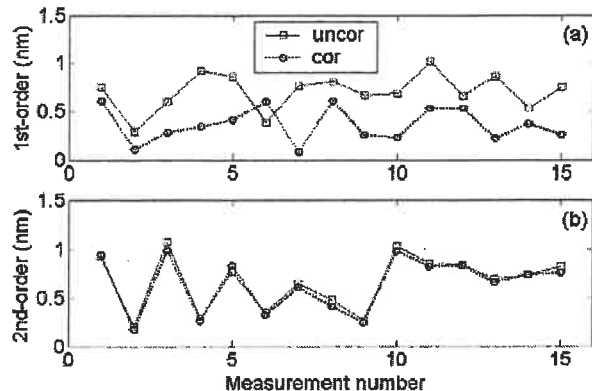


Figure 8: Repeatability testing results for nominal orientations of half wave plate and polarizer: a) first-order error amplitudes; and b) second-order error amplitudes.

5. Conclusions

Experimental validation of the digital first-order periodic error reduction scheme described by Chu and Ray [1] was completed using a bench-top setup of a single-pass, heterodyne Michelson interferometer. The strategy was to minimize common error contributors such as Abbe, dead path, and environmental errors through the setup design in order to isolate periodic error. Linear, oscillating motion generation was accomplished using a parallelogram, leaf-type flexure. The setup also enabled variation of the periodic error through independent rotation of a half wave plate and polarizer. Experimental results demonstrated that the correction algorithm can successfully attenuate first-order error to sub-nm levels for a wide range of frequency mixing conditions.

Acknowledgements

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Synchronous Acoustooptic Tuning of Free Space External Cavity Lasers for Homodyne Chirp Interferometry

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1. Introduction

The frequency ramped continuous wave (FMCW) technique for distance ranging originated as a radar technique. More recently, its optical counterpart has been developed for absolute laser interferometry. However, a major disadvantage of this technique has been its lack of accuracy, due to the limited chirp rates available with the laser sources that have been used. The most commonly used source has been a semiconductor laser diode, tuned by varying its injection current. In this case, the sweep range is limited by mode hopping, and the sweep rate is limited by thermal effects. Mechanically tuned external cavity lasers have also been used as sources. These have much greater sweep ranges, but because the tuning is mechanical, their sweep rates are slow. Here, the use of acoustooptically tuned external cavity lasers is proposed, since these have the potential of sweeping rapidly over wide tuning ranges without mode hops. The resulting performance gains are estimated, and a brief description of a possible design is outlined. (A paper describing the design and some preliminary proof of concept tests is published elsewhere.)

The principle of the FMCW method is to mix the light beam returned from the target reflector with the beam emitted by the source: the frequency of the resulting beat frequency is then a measure of the distance (x) between the source and the target. If the speed of light is c and the chirp rate of the beam is $\dot{\nu}$, then the beat frequency is

$$f = \frac{2\dot{\nu}x}{c}. \quad (1)$$

Kubota et al [1] proposed and demonstrated an early version of such a scheme, which had a range of a few metres, limited by the coherence length of the source. This was a 0.79 μm (380 THz) laser diode, and its injection current was varied in order to generate a frequency ramped output beam: the modulation of optical frequency was about ± 30 GHz at a modulation rate of 90 Hz. This corresponds to a chirp rate of about 2.7 THz/s. At a range of 1 m, the beat frequency would therefore have been 18 kHz.

More recently, Minoni et al [2] reported on a low cost, compact and robust implementation of a system based on the same principles. In this case, the beat frequency was about 52 kHz for every metre of range. They reported that the frequency measurement resolution was 3.5 Hz, giving a spatial resolution of about 70 μm .

Both these systems used semiconductor laser diodes as sources, chirped by varying the injection current. This type of chirped source has two disadvantages. Firstly, the optical frequency can be modulated by no more than a few tens of GHz before mode hopping occurs (and the effect of a mode hop would be catastrophic). Secondly, the modulation rate is limited to no more than a few hundred Hz by the thermal time constant of the diode. As a result, the chirp rate can be no more than about 10 THz/s, so the beat frequency can be no more than a few tens of kHz at a range of 1 m. To increase this beat frequency, a wider range of modulation frequency and/or a faster modulation rate are required to generate a faster chirp.

The mechanically tuned Littman design [3] is well established as the basis for many phase continuously tunable external cavity lasers. (The phrases “phase continuously tunable” and “mode hop free” are synonymous.) Its key attribute is that it has a wide tuning range: Stone et al [5] used a laser of this type as the source for an absolute interferometer. Although the tuning range can be very wide, the sweep speed is limited by the mechanical drive. For example, a commercial product [4] can tune, free of mode hops, from 1440 to 1640 nm, an optical frequency range of 25 THz. However, the mechanical tuning limits the maximum sweep speed to 80 nm/s, which corresponds to a chirp rate of 10 THz/s. (Incidentally, the device used by Stone et al had a sweep speed of only 8 nm/s.)

The elimination of moving parts from such an external cavity laser would overcome the sweep speed restriction, and in addition would remove motor vibration and increase alignment stability. Acoustooptic tuning is an attractive way of increasing the sweep speed by at least two orders of magnitude. Moreover, the lasing wavelength is controlled electronically, allowing chirps of very high linearity to be generated.

Although acoustooptic tuning of lasers is well established, mode hop free (continuous phase) tuning over wide frequency ranges has not in the past been achieved. However, a scheme for doing this has recently been proposed, and its feasibility has been demonstrated experimentally [6]. This type of source is capable of ramping at 1000THz/s or more, and could significantly improve the performance of absolute interferometers.

This abstract is organized as follows: Section 2 discusses the relationship between chirp rate and accuracy; Section 3 describes how an interferometer of this type could be used not only for ranging, but tracking a moving object; Section 4 summarizes the principle of the acoustooptically chirped laser.

2. Accuracy

From equation (1) it follows that the error in range Δx resulting from an error in beat frequency measurement Δf is

$$\Delta x = \frac{c}{2\nu} \Delta f. \quad (2)$$

The accuracy with which Δf can be measured varies inversely with the measurement time T (which may extend over more than one frequency sweep). It is also to be expected that the better the signal to noise ratio R of the beat signal, the more accurately can its frequency be measured. An approximate estimate derived in the Appendix suggests that

$$\Delta f = \frac{1}{2TR^{2/3}}. \quad (3)$$

As an example, assume a source with a chirp rate of 5 THz/s, a measurement time of 10 ms and a signal to noise ratio (SNR) of 125. Then the accuracy with which the beat frequency can be measured is 2 Hz, and the range accuracy is 60 μm .

Now assume a source with a chirp rate of 500 THz/s. The bandwidth of the electronics used to measure the beat frequency will now need to be increased by a factor of 100, and it is assumed that consequently, the noise will increase by a factor of 10, reducing the SNR to 12.5. Again assuming a measurement time of 10 ms, the beat frequency can be measured to about 9 Hz, and the range to about 3 μm .

These examples are fictitious, and they depend on a number of assumptions. Nevertheless, they do indicate that significant range accuracy improvements could be obtained by using faster chirp rates.

3. Tracking a moving object

So far, it has been assumed that the interferometer is used for ranging a stationary object, and equation (1) is written on that basis. However, it is, at least in principle, possible to use this system for tracking a moving object. Imagine that the object is moving away from the source at a velocity $\dot{x}$. Then a Doppler shift term must be added to the right side of equation (1), which becomes

$$f = \frac{2\nu\dot{x}}{c} + \frac{2\dot{x}\nu}{c} = \frac{2}{c} \frac{d}{dt}(\nu x) \quad (5)$$

where t represents time. Now assume that at a time t_1 , the instantaneous optical frequency is ν_1 and that the range is known to be x_1 . At a later time t_2 , the optical frequency is ν_2 . During this time, N_{12} beat cycles have occurred. By integrating equation (5), it can be shown that the range at t_2 is given by

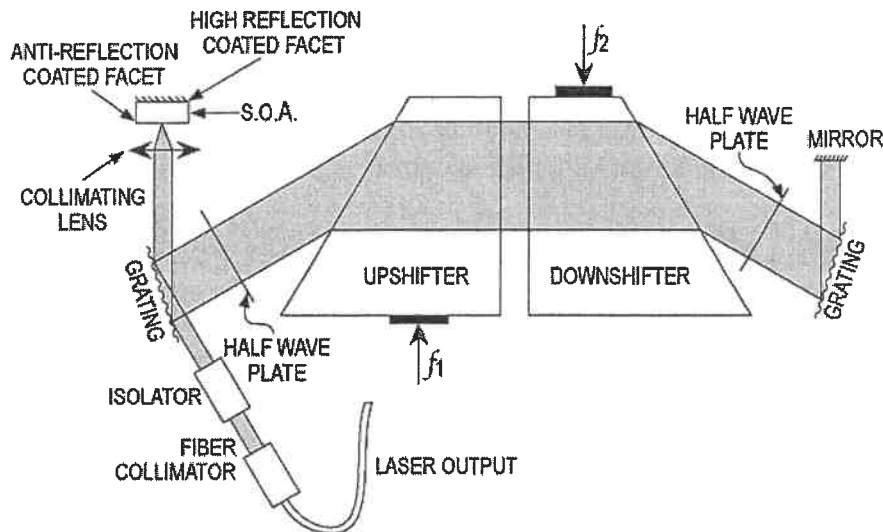
$$x_2 = \frac{\frac{c}{2} N_{12} + x_1 \nu_1}{\nu_2}. \quad (6)$$

To avoid a sign ambiguity in frequency when the object is moving towards the source, its speed must not exceed $x\dot{\nu}/\nu$. At a range of 1 metre, a frequency of 200 THz and a chirp rate of 500 THz/s, this critical speed is 2.5 m/s.

4. Acoustooptically synchronously tuned laser

A vital attribute of the Littman laser design is its ability to tune without discontinuities of phase. A particular geometrical configuration is adopted which ensures that as the cavity is tuned by changing its length, the pass band of the mode filter is tuned in exact synchronism with the cavity resonance. It is this synchronous tuning of the lasing mode and the mode filter that allows phase continuous tuning to be obtained over large frequency ranges.

Until now, no corresponding form of synchronous tuning has been demonstrated for acoustooptically tuned lasers. However, a simple condition for synchronous tuning in such devices has now been derived. The cavity in the figure below is designed on this basis. The tuning elements are the two acoustooptic devices, one configured as an upshifter and the other as a downshifter. To generate a linear chirp, these are driven by two acoustic frequency ramps, and it can be shown that if the time separation between these ramps is chosen correctly, synchronous tuning will result. (The required time separation is equal to one half of the product of the cavity's round trip transit time and the rate of change of the mode filter's optical pass frequency with acoustic frequency.)



An acoustooptically tuned laser design which is capable of phase continuous tuning over wide frequency ranges. Gain is provided by the semiconductor optical amplifier (SOA) and the cavity is tuned by the acoustooptic deflectors (labeled “upshifter” and “downshifter”) in conjunction with the diffraction gratings. The RF inputs to the deflectors (f_1 and f_2) are both ramps, but slightly displaced in time. When this time displacement is appropriately chosen the laser emits a phase

continuous chirp over a wide frequency range. The output is derived from the specular reflection at the first grating. The waveplates simplify the mechanical design by allowing the laser to be constructed on a flat baseplate.

This particular design has not been built, but a simpler version has been built and successfully tested to prove the principle of operation. In particular, it tuned synchronously in the near IR (about 1550 nm). Further details are given in reference [6].

5. Summary

The principle of FMCW absolute interferometry is well established. However, the accuracy of this technique has so far been limited by the chirp rate of the sources available. A significant improvement could be made by using a continuously tunable (mode hop free) acoustooptically tuned external cavity laser as the source. The chirp produced by this type of source can be made highly linear, which also improves accuracy. The FMCW technique is capable not only of ranging stationary objects, but also of tracking moving ones.

Appendix

Assume that the sinusoidal signal whose frequency is to be measured can be represented as $V_1 = \cos 2\pi ft + n(t)$ where the second term represents noise. Define the signal to noise ratio, R , as the reciprocal of the root mean square of the noise term, that is $R = \langle n^2(t) \rangle^{-1/2}$. The SNR is assumed to be significantly greater than 1. A simple method of measuring the frequency would be to count the number of zero crossings in a time T . However, the precision of this method would be limited because only complete half cycles were being counted.

Better precision could be obtained by frequency doubling the signal before counting the number of zero crossings. Successive frequency doublings would further increase precision. However, each time the frequency is doubled, the signal to noise ratio is reduced, and eventually it will fall to below 1. At this point, it is assumed that there is no point in carrying out further doublings. If the number of doublings carried out is N , then the precision of the frequency measurement will have been increased by the factor 2^N .

Assume that frequency doubling is accomplished by squaring the signal. Thus, the original signal becomes

$$V_1^2 = [\cos 2\pi ft + n(t)]^2 = \frac{1}{2} + \frac{1}{2} \cos 2\pi(2f)t + 2n(t) \cos 2\pi ft + n^2(t)$$

The second term on the right is the new signal at double the frequency of the original, and its amplitude is half of that of the undoubled signal.

The third and fourth terms represent noise. It is assumed that the third term dominates the fourth one, which is therefore ignored. (This will be valid only if the signal to noise ratio is significantly greater than 1. However, since the doubling process is truncated when the ratio is equal to 1, the error introduced is small.) The root mean square of the third term is

$$\langle 4n^2(t) \cos^2 2\pi ft \rangle^{1/2} = \langle 4n^2(t) \left[\frac{1}{2} + \frac{1}{2} \cos 2\pi(2ft) \right] \rangle^{1/2} = \sqrt{2} \langle n^2(t) \rangle^{1/2}.$$

Thus, every time the sinusoid is doubled, the amplitude of the signal is halved, and the noise is increased by a factor of $\sqrt{2}$: thus the SNR is decreased by the factor $2\sqrt{2}$. Hence, the number of doublings required to reduce the SNR of the original signal (which was R) to 1 is equal to $N = \frac{2}{3} \log_2 R$. The corresponding frequency is

$f \times 2^N = f \times R^{2/3}$. In the measurement time T , the number of zero crossings of a sinusoid of this frequency is $2fTR^{2/3}$. The relative uncertainty in measuring this frequency by counting zero crossings is $\left(2fTR^{2/3}\right)^{-1}$ and so the frequency error is $\Delta f = \left(2fTR^{2/3}\right)^{-1}$.

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Deflectometry Rivals Interferometry

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Interferometry can measure the distance of atomic layers and the distance of stars. This remarkable dynamic range of more than 24 orders of magnitude contrasts with its weakness to measure strongly aspheric surfaces or deep surface steps.

To overcome the difficulties of aspheric surface measurement is one motivation of the paper. Surprisingly, we found a solution for this problem by asking a completely different question: "how much may an optical 3d-sensor cost?"

To answer the latter question it is appropriate to consider an optical sensor as a communication system [1]. A canonical communication system has an information source, a source encoder, a channel, a decoder and a sink. The identification of the sensor components to the components of the canonic system is crucial. Channel capacity (amount of information that can be transmitted through the channel) is expensive. We can define an "information efficiency" of an optical sensor: the more raw data we have to acquire, for the shape of the object, the lower is the efficiency and the more costs the sensor.

The efficiency is quite different: for stereo vision ~ 1%, for fringe projection ~ 12%, for white light interferometry ~ .1%, for shape-from-shading ~ 50%. The reason is that different sensor principles include more or less source encoding. What is the source encoder in our optical sensors? It is the illumination system!

What should the illumination system do, for efficient sensing? In practice, all objects are highly correlated, their power spectrum drops with $1/f$ (f =spatial frequency) – they display much redundancy. A proper source encoder should equalize the object power spectrum. A nice approximation to do this is optical differentiation, so we need an optically differentiating optical sensor: For diffusely scattering objects "shape-from-shading" is the proper principle. For smooth objects it is shearing interferometry, Nomarski differential interference contrast, or "deflectometry" [2].

Estimating these systems we find that deflectometry reveals some remarkable features – making it a rival of interferometry: It intrinsically measures the local slope of the surface. So the acquisition of the local refractive power does not require a (noise-prone) second order derivation. The slope may range from -60° to $+60^\circ$ or more. The sensitivity against small slope variations ranges down to less than 1/100 arc sec. The high sensitivity against local "defects" is remarkable. We can resolve a few nanometers depth variation over only 1 mm lateral distance, working at Heisenberg's limit. The object may have

several meters size (car wind shields). The total shape accuracy is limited by the calibration accuracy and by the numerical integration procedure. We could achieve an accuracy of a few micrometers over 100 mm object size.

Eventually, I will mention one last advantage, which makes deflectometry robust against environmental perturbation: it is not based on interferometry.

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Application of Phase Retrieval to Precision Optical Metrology

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Phase retrieval is a non-interferometric technique for measuring a wavefront. Its main distinguishing characteristic is the simplicity of the apparatus required to make a measurement <96> only a known illuminating field, an optical surface or system under test, and a detector array are required. The detector array measures intensity patterns in one or more near-focus planes formed by the illuminating field after it has interacted with the surface or system under test and no additional optics are required. Phase retrieval is vibration-tolerant and capable of measuring aspheric wavefronts without having to account for retrace errors that occur in interferometry, simply because there are no optics through which the deviated rays must travel.

The complexity of the technique lies in the phase retrieval algorithms used to determine the wavefront in the exit pupil from these intensity patterns. They are iterative in nature and must accurately model the physical optics propagation from the exit pupil to the measurement planes.

The technique has been used previously in wavefront sensing for adaptive optics [1], to determine the fabrication errors in the Hubble Space Telescope [2], and will be used for fine phasing of the segmented primary mirror of the James Webb Space Telescope. However, using phase retrieval in the manufacture of precision optics is a demanding new application.

Modeling results, including the effects of realistic detector noise, have shown that phase retrieval can achieve accuracies of better than $\lambda/200$ (RMS) using two measurement planes and of $\lambda/1000$ using a larger number of planes. We anticipate that real-world effects may degrade this extreme accuracy. The limits on accuracy and how to surpass them are a major topic in our research.

Accurate positioning of the detector is important, but a refinement in the algorithm can greatly reduce this requirement. This is an example of a situation where complexity in an experimental arrangement can be eliminated by increasing the complexity of the phase retrieval algorithm.

We have also begun to explore the limits of the capabilities of phase retrieval to measure different types of surfaces and system. The speed of the beam (NA or f-number) incident on the detector is limited by the detector pixel spacing. The slope of the deviation of the wavefront from spherical, which limits the asphericity that can be measured, is limited by the number of pixels in the detector. A promising technique for extending the range of optics that can be measured is to make undersampled measurements of the intensity patterns [3], although determining the uniqueness and accuracy of the wavefront estimate under that circumstance requires further work.

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The Heterodyne Profiler Redux

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Keywords: heterodyne profiling interferometer, HPI, surface roughness

Introduction:

The heterodyne profiling interferometer is a surface roughness measuring laser interferometer originally designed by Gary Sommargren in the early 1980's. The original prototype was built at Lawrence Livermore National Laboratory by Gary and his colleagues at Livermore. After Gary joined Zygo Corporation, Zygo developed and marketed a commercial version of this instrument under the name HPI (Heterodyne Profiling Interferometer). The original Zygo instrument was able to measure surface roughness to less than 1 angstrom, with a noise floor of about 1/10 angstrom. Due to the elegance and simplicity of Gary's optical design, the instrument is still used today, as the only surface roughness instrument that can achieve such high resolution and low noise.

In 2003, Convergent Technologies began a program to refurbish and update an HPI belonging to Zygo Corporation. While the system electronics, controls and computer were obsolete, the strength of the basic optical design warranted a complete redesign and replacement of all the supporting subsystems. The refurbished instrument continues in regular use in Zygo's measurement lab.

This paper outlines our experience in rebuilding this historic instrument. We will illustrate the changes in electronics and computers that have occurred in the 25 years since the HPI was developed. As a tribute to Gary Sommargren, we will illustrate the elegance of Gary's original optical design, which is still providing crucial surface roughness measurements, a quarter of a century after its invention; long after all the other parts of the original HPI instrument have become obsolete.

The Heterodyne Profiling Interferometer:

The basic optical design was devised by Gary Sommargren about 1980 (refer Optical Society of America, Applied Optics / Vol. 20, No. 1 / 15 Feb 1981). Gary described a non-contact optical technique for surface profiling with a height sensitivity of the order of 1 angstrom.

Zygo productized the instrument and sold the first commercial product in 1987. The original HPI included a large, 19" rack-mount cabinet to contain the electronics and computer necessary to run the instrument. Critical optical adjustments for focus, illumination intensity, and starting optical phase were adjusted manually with knobs protruding from the cover.

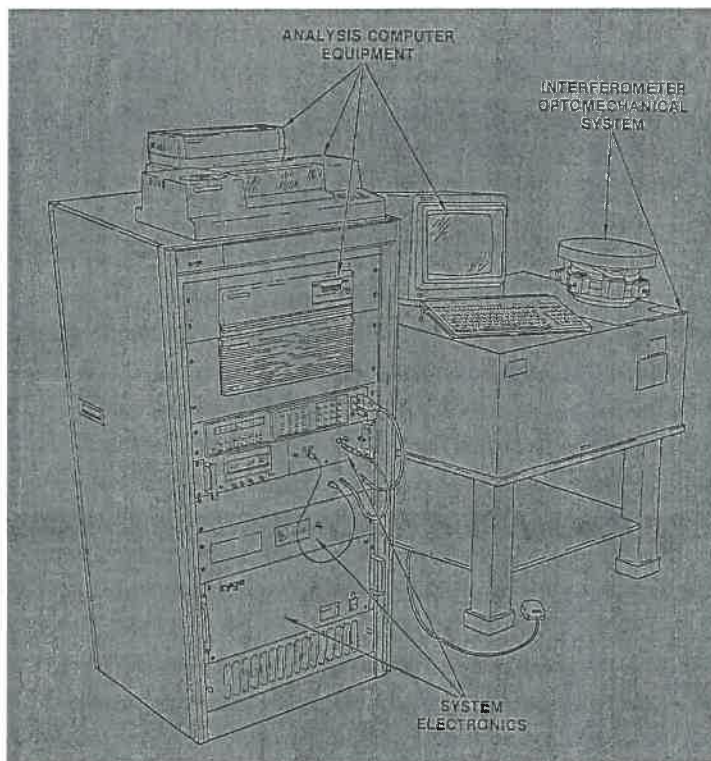


Figure 1: an illustration of the original HPI from the HPI manual (circa 1987). A 19" rack cabinet contains computer, phase meter and electronics necessary to run the HPI.

Focus was adjusted by reaching through an access door on the front of the machine and adjusting a micrometer while observing the light pattern on a special glass target.

Illumination intensity and starting phase of the interferometer were adjusted with control knobs protruding from the rear of the instrument.

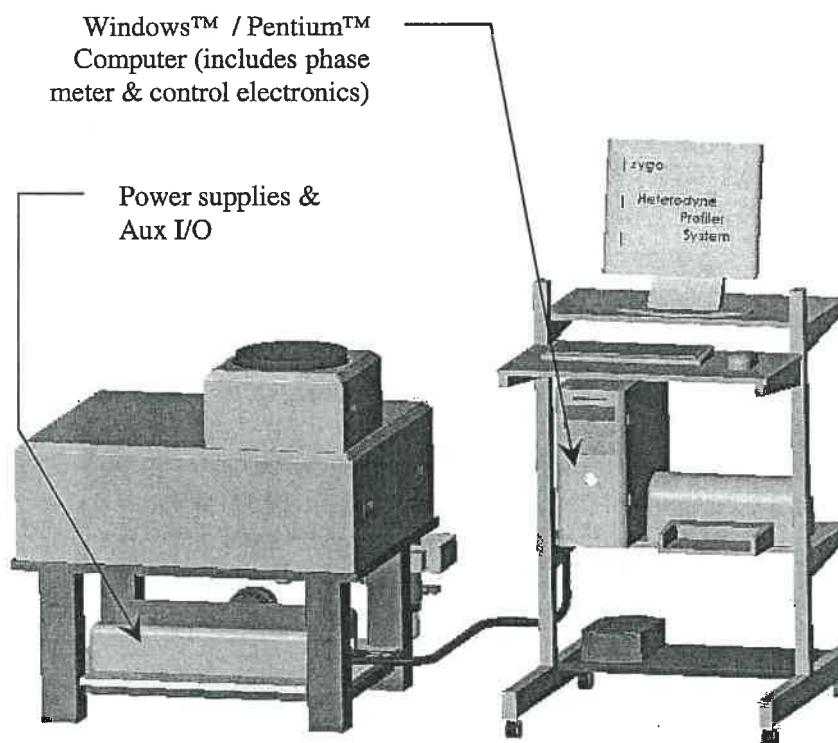


Figure 2: The refurbished HPI, completed in 2003.

The entire electronics rack was discarded, replaced with the Windows™ Pentium™ computer, add-on boards plugged into the PCI slots, and a custom software application.

Only Sommargren's original interferometer design was retained.

In 2003, Convergent Technologies undertook a project to refurbish an HPI which had been mothballed and placed in storage at Zygo Corporation. During the course of this program, we scrapped all the electronics. We redesigned and simplified the optical system and automated what were previously manual adjustments. Due to advances in computers and electronics in the last two decades, we eliminated an entire rack of electronics (22" x 34" by 50" high, weighing about 300 lbs) with a Windows™ Workstation computer with a special A/D board installed in the PCI slot.

During the process of redesigning and rebuilding the instrument, we were able to see many areas where the technology has advanced by leaps through the 1980's and 1990's.

In light of today's computers, original HP-300 computer that controlled the HPI is little more than a programmable computer. A significant part of the HPI operations manual was devoted to explaining how to work the computer. The refurbished instrument is controlled by a standard Window™ / Pentium™ workstation. The user interface is standard Windows™ look and feel. Operators are assumed to understand this interface intuitively.

The original phase measurement electronics consisted of a special electronic phase instrument and an HP Digital Multimeter to read the analog voltage output from the phase meter. This was replaced with a high performance A/D board plugged into the PCI slot of the computer and custom software which runs special DFT (discrete Fourier transform) calculations developed by Zygo for their current products.

About a mile of various types of wire and coax cable, in proprietary cable harnesses, was replaced by one RS-232 cable and one USB cable running between the HPI and its computer.

The optical system was redesigned and simplified. In addition, focus and intensity adjustments were automated. A "starting phase" adjustment, required due to limitations in the original phase meter was eliminated. However, the basic optical arrangement of interferometer was retained (see figure 3). This is a testament to elegance of Gary Sommargren's original interferometer design. After replacing broken optics and repairing the laser, the interferometer made surface roughness measurements of parts in the 2 angstrom range with repeatability well under 1/2 angstrom.

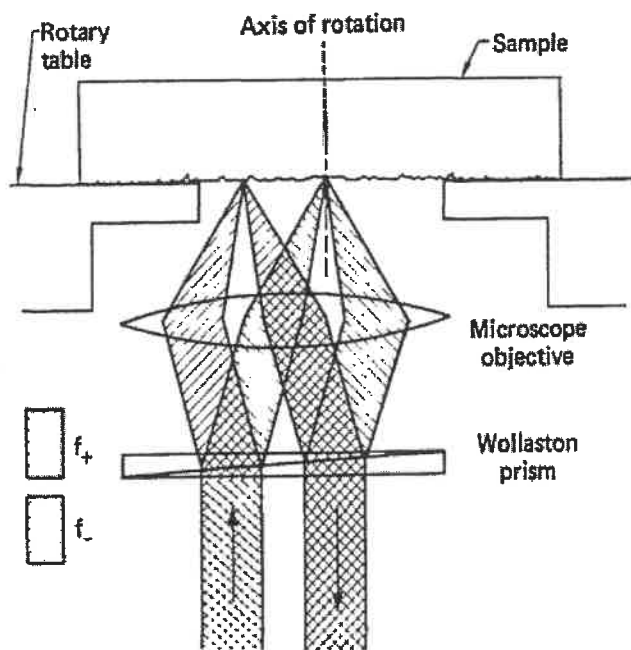


Figure 3: Illustration of interferometer optics described by Sommargren in 1980.

A two frequency, polarization coded, HeNe laser beam is split into two paths by a Wollaston prism and focused onto two spots on the sample by a microscope objective.

Light reflected from the two spots on the sample is collimated by the microscope objective and recombined by the Wollaston prism. The sample being measured is rotated so that one of the two illuminated spots lies on the axis of rotation.

Acknowledgements:

This paper is dedicated to Gary Sommargren, who spent most of his professional career making important contributions to the science and practice of interferometry. The HPI optics described here are simply one example of his work. I would also like to acknowledge the helpful discussions and advice of Ned Bennett, Peter De Groot, Frank Demarest, Chris Evans, Bob Smythe, and Carl Zanoni.

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Rapid, accurate, and high-resolution metrology of aspheric optics — can we “have it all”?

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Performance requirements for optical metrology systems are strongly application dependent. In fact, since the advent of deterministic finishing, metrology is integrated even more tightly into optics fabrication. Two types of recurring questions now drive some of these requirements: (i) With this metrology, can I be confident in determining whether a part meets the stated spec? and (ii) Can I reasonably expect to improve the part by using this metrology as input for an additional cycle of finishing? The latter question has the added requirement that quantified maps of figure error must have an explicit correspondence between points in the map and all points on the test surface. Increasingly, the former proceeds by reference to the power spectrum density (PSD) of the surface's deviation from its desired nominal shape. This spectrum spans from figure/form through waviness/mid-spatial frequency and down to roughness (although roughness is generally treated separately). As optical tolerances tighten in high-end applications, and as designs require stronger aspheres, metrology often ends up on the critical path in terms of meeting specifications and schedules. This puts an increasingly heavy emphasis on characterizing metrology performance and understanding the real requirements. These tasks are by no means trivial. Nevertheless, the question posed in the title can be addressed in general terms.

In terms of accuracy, resolution, and speed/versatility, interferometric systems have uniquely met the requirements for the metrology of spherical optics. Speed is the chief sense in which these systems are not consistently stretched by current engineering requirements; these systems are oftentimes inadequate in terms of their aperture capabilities (both clear aperture and numerical aperture) and especially in their ability to deal with aspheres. While aspheres can be addressed by using dedicated nulls or coordinate measuring machines, such approaches come at the cost of one or more of speed/versatility, resolution, or accuracy. A natural option for trying to have it all is to supplement the existing workhorses with significant new capabilities.

Every decade or two, we are handed a factor of a thousand in terms of added computing power: gigaflop processors coupled with gigabytes of RAM and hard disks holding hundreds of gigabytes are now everyday. By coupling this new power to high-precision computer-controlled machines, it would be surprising if we could not greatly amplify the capabilities of a variety of optical metrology systems that gather megapixels of data at each shot. Further, since speed is typically not a key limitation, it should be workable to gather a variety of complementary data sets and fuse them via a range of possible numerical processes. In particular, we present new ideas and results to demonstrate that the old notion of data stitching is ripe for exploiting these new capabilities. In fact, stitching can simultaneously improve the performance of metrology systems in each and every metric mentioned above. It is only in that sense that we can hope to fleetingly have it all. The good news for those who relish a challenge is that, for the foreseeable future, at least the goal posts for accuracy and aspheric departure are sure to keep moving beyond whatever is delivered.

Key words: asphere metrology, stitching interferometry

Minimizing Uncertainty in Cryogenic Surface Figure Measurement

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A new facility at the Goddard Space Flight Center is designed to measure with unusual accuracy the surface figure of mirrors at cryogenic temperatures down to 12 K. The facility is currently configured for spherical mirrors with a radius of curvature (ROC) of 600 mm, and apertures of about 150 mm or less. The goals of the current experiment were to 1) Obtain the best estimates of the cryo-changes, $\Delta(x,y)$: the changes in surface figure between room temperature and the two cryo-temperatures 87 K and 20 K; and 2) Determine the uncertainty of these measurements, using the definitions and guidelines of the *ISO Guide to the Expression of Uncertainty in Measurement*.

A silicon carbide mirror was tested, and the cryo-change from room temperature to 20 K was found to be below 4 nm rms. This report describes the facilities, experimental methods, and uncertainty analysis of the measurements.

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Enhancing Measurement Accuracy with Subaperture Stitching Interferometry

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Keywords: interferometry, stitching, absolute testing, surface figure measurement

Interferometry has long been the method of choice for measurement of optical surfaces. With proper environmental control, nanometer-level precision and beyond can be achieved. Systematic errors (biases), however, increase the measurement uncertainty beyond the precision. A major source of such error is the quality of the reference wavefront. In typical commercially available transmission spheres, the reference wavefront is imperfect by 50 nm or so PV (over its full aperture) – 1 to 2 orders of magnitude higher than the precision!

Several methods exist for calibrating this bias, and thereby narrowing the gap between uncertainty and precision. They employ the principles of error reversal and/or averaging to separate the surface measurement from errors in the reference wavefront. As a result, these methods all require multiple measurements under different orientations of the test and reference parts. Examples include the three-flat, two-sphere, N-position, and random-average tests. Subaperture stitching provides an alternative method of reference wave calibration. Most stitching methods rely on pre-calibration of the reference wavefront (using one of the aforementioned methods), but QED's technique (patent pending) allows the reference wave bias to be calibrated as part of the stitching computation. Reference wave bias causes a discrepancy in regions of subaperture overlap, which can be modeled and thus compensated by the algorithm. This eliminates issues of calibration "lifetime" or applicability, as the reference wave is computed in-line with the stitching.

In theory, any of these methods can capture the reference wave bias. Yet naturally all the methods have limitations in precision and lateral resolution that require certain trade-offs to be made. While the stitching algorithm can achieve any resolution in principle, the robustness of the calculation and the available computing power typically limit the practical use to a few hundred Zernike polynomials. The two-sphere method provides a full resolution estimate; but suffers from alignment sensitivity and a poor cat's eye measurement. Furthermore, the two-sphere test is not always possible, since the cat's eye can only be measured when the point focus of the transmission sphere is accessible to the stage travel. The random-average method requires a calibration artifact, can be quite time consuming, and suffers from low-order inaccuracy if the calibration artifact is not a complete sphere.

Stitching does, however, have one key advantage over all the others – it can be used in tandem with any other method. For example, a random average or two-sphere reference can be computed and subtracted from the subaperture data before it is stitched. The stitching algorithm can then optimize over any remaining systematic error (for example, originating from inaccuracies of the calibration method or time separation

between the calibration and the actual measurement). This provides the best of both worlds: a full resolution calibration (for removing mid-spatial frequencies of the reference wave) and an accurate low order calibration that is not time-separated from the data. We demonstrate results of various calibration methods on QED's subaperture stitching interferometer (SSI[®]), and show the advantages of using stitching in tandem with other methods.

Customized three-flat calibration method for a 320 mm aperture Fizeau interferometer with vertical optical axis

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We describe a modified version of the traditional three-flat calibration method, which we have used to calibrate the 320 mm diameter reference flat of the Large Aperture Digital Interferometer (LADI) at the CSIRO's Australian Centre for Precision Optics. The reference flat has a Clapham-Dew type coating and is subject to gravitational sag/distortion. The salient features of the new calibration method are a combination of proper sequencing of pairs of flats, rotational averaging and utilization of symmetry properties to infer absolute surface relief data. We present the data acquisition and processing schemes and show the results of the first experimental calibration run. Due to residual thickness variations of the Clapham-Dew coating, which are picked up by the transmitted object wave, there is a distinct constant instrument function which requires an extension of the standard method. An additional refinement is necessary to account for practical positioning uncertainty. The eventual calibration results are expected to validate the method and establish an accuracy better than 1.5 nanometers rms.

GAUSSIAN BEAM MODELING OF THE RADIUS OF CURVATURE

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Abstract:

A Gaussian model of the radius measurement of micro-optics has been developed and tested using simulations. The model is based on the propagation distances in the interferometer, a heretofore uninvestigated effect. The goal of the model is to determine the bias error in the radius due to the Gaussian Beam propagation model. After testing the model with varying conditions, we have concluded the following: the measured part is smaller than the input, the cat's eye and confocal positions have approximately the same error, radius error increases with smaller test parts, decreasing the numerical aperture increases the errors, and the propagation distances do not affect the radius. The outline of the experimental plan to be used to verify the results is given.

1.0 Introduction

Micro-optic components are the key to building compact optoelectronic systems and are used in many applications such as optical networks, medical imaging, and optical data storage. These components have aperture diameters from 10 micrometers up to fractions of a millimeter. A crucial step in the manufacturing of the refractive micro-optic lenses is the measurement of the radius of curvature (ROC). The interferometric measurement of ROC is defined as the distance between the confocal reflection and the cat's eye retro-reflection [1, 2, 3]. The confocal position occurs when the optic is placed in the test beam of the interferometer such that the wavefront curvature of the incoming beam and the curvature of the test optic match. In the geometric model, the cat's eye position occurs when the surface of the optic is coincident with the focus of the test beam.

Traditionally, a geometric model of each position is used to describe the ROC measurement [1]. For micro-optics and high precision macro-optics, this model is not adequate to describe the ROC measurement. We believe a Gaussian beam propagation model is required and will offer lower ROC measurement uncertainty due to eliminating a systematic bias. This paper describes Gaussian beam approximations used to model the radius measurement and proposed experiments to validate the model.

2.0 The Radius Measurement

A schematic of the radius measurement is shown in Figure 1. The laser and collimating optics (not shown) produce a collimated beam with a well defined waist that enters the beam splitter where the light is split into two arms, the reference arm and test arm. In the reference arm, the beam propagates a distance of d_r , reflects off the reference mirror (labeled Ref. mirror) and again propagates a distance of d_r . In the test arm, the beam propagates a distance of d_t , is focused by a lens with focal length, f , propagates a distance s , reflects from the test optic with radius r , propagates a distance s , is collimated by a lens with focal length, f , and is propagated a distance d_t . At the beam splitter, the test and reference arms interfere and are focused onto the camera using imaging optics (not shown). Note that in Figure 1, the ray traces are shown to demonstrate the operation of the interferometer and do not indicate the Gaussian beam.

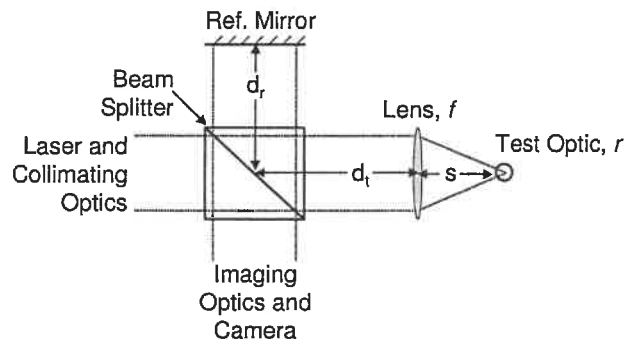


Figure 1: Radius measurement schematic.

3.0 The Gaussian Beam Model

A Gaussian beam can be described with a complex beam radius of curvature, q [4]:

$$\frac{1}{q} = \frac{1}{R} + i \frac{\lambda}{\pi w^2}$$

Equation 1,

where R is the radius of curvature of the wavefront and w the beam half width. The parameters R and w , and consequently the complex beam radius, are functions of position along the optical axis. That is, as the beam propagates in space (or through an optical system) the beam curvature and size change. The complex beam radius and the propagation of the beam through the optical system are used to model the radius measurement.

We can identify an input complex beam curvature as q_{in} (defined later). As the beam passes through each element in the optical system, the complex beam curvature changes. Each element is represented by a 2x2 matrix, M , in the traditional manner [5], as shown in Equation 2. The output beam complex radius of curvature, q_{out} [4], can be calculated in the following manner:

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad q_{out} = \frac{Aq_{in} + B}{Cq_{in} + D}$$

Equation 2.

3.1 The Matrices

This section defines the matrix, M , which is used to model the interferometer. Each element in the test and references arms can be described using the matrix method of analysis. These elements are propagation, mirrors, and lenses. Equation 3 gives the equations for propagation through a distance d , M_{Prop} , a thin lens with focal length f , M_{Lens} , a mirror with radius r (the test optic), M_{Optic} , and the reference mirror (radius is infinite), M_{Mref} [5].

$$\begin{aligned} M_{Prop} &= \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} & M_{Lens} &= \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \\ M_{Optic} &= \begin{bmatrix} 1 & 0 \\ 2/r & 1 \end{bmatrix} & M_{Mref} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Equation 3.

Each arm of the interferometer is then represented by the multiplication of the matrices

of each element in the arm. The reference arm matrix, M_{ref} , is propagation, reflection, and propagation as shown:

$$M_{ref} = \begin{bmatrix} 1 & d_r \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & d_r \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2d_r \\ 0 & 1 \end{bmatrix}$$

Equation 4.

The matrix for the test arm, M_{test} is more complex with propagation by d_t , focusing by f , propagation by s , reflection at r , propagation by s , collimation by f , and propagation by d_t as shown:

$$M_{test} = \begin{bmatrix} 1 & d_t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2/r & 1 \end{bmatrix}^* \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & d_t \\ 0 & 1 \end{bmatrix}$$

Equation 5.

The M_{ref} and M_{test} matrices are simplified and used in Equation 2 to calculate the output beam complex radius of curvature of the reference and test beams respectively, q_{rout} and q_{tout} . The parameter q_{tout} is a function of the position s (the propagation distance after the lens) while q_{rout} is constant and depends on the interferometer configuration.

4.0 The Confocal and Cat's Eye Positions

The confocal and cat's eye positions both occur when q_{rout} equals q_{tout} , that is, the interference pattern is flat. The reference arm q_{rout} is calculated using Equation 4 and Equation 2, and the test arm q_{tout} is calculated using Equation 5 and Equation 2. Since q_{tout} is a function of s and q_{rout} is constant there will be two positions where q_{rout} equals q_{tout} , each corresponding to a different value of s , one defining the cat's eye position and one defining the confocal position

The radius is then calculated as the difference between the cat's eye and confocal. It should be noted that this method requires defining q_{in} , the focal length of the lens, and the propagation distances, d_t and d_r . The nominal values for these parameters are based on the experimental setup. They are varied in simulation to study their effect on radius.

It should be noted that the propagation distances, d_t and d_r , can be set to provide for a perfect cat's eye and confocal reflections. However, the

propagation distances for the perfect cat's eye are not the same as for a perfect confocal. Obviously, moving either the reference mirror or the objective lens during the measurement is not feasible. Therefore a bias is introduced into the measurement. In addition, many of the required distances for perfect reflections are not practical (e.g. greater than 1 meter for a 100 μm part). Also, a precise measurement of the actual distance of the reference mirror and/or the lens can be prohibitively costly. Therefore, we will set the reference mirror and lens locations in the model using approximate values from our interferometer. We will study the effect of the position of these parts on the radius.

5.0 Determining the Inputs

The goal of this algorithm is to determine the error in the measured radius which changes with varying inputs like NA and part radius. The NA is changed in this model by varying the input beam complex radius of curvature, q_{in} . In the sections below, we describe two cases to calculate q_{in} and then the calculation of each of the remaining inputs.

5.1 Calculating q_{in} , Case 1

First we consider a standard interferometer with an aperture stop immediately before the beam splitter. The input beam is assumed to be collimated at this point and the aperture stop is overfilled such that the real part of the complex curvature goes to zero, that is, the beam is assumed to be flat at the stop. The complex q_{in} [4] for this case is

$$q_{in} = i \frac{\pi w^2 n}{\lambda} \quad \text{Equation 6,}$$

where w is the radius of the aperture stop, n the index of the medium (assumed to be 1) and λ is the wavelength of the incident light (a Helium-Neon laser at 632.8 nm). For an 8 mm diameter aperture stop (approximately equivalent to our micro-interferometer), q_{in} is $i*79393.17$ mm.

5.2 Calculating q_{in} , Case 2, and focal length

Another method to determine the input beam curvature is to work backwards from the focus point. This method assumes that the system is well designed and has a diffraction limited focus point. The inputs to this are the numerical aperture, NA, and the input beam diameter. The waist of a Gaussian beam at focus, w_0 , and the complex curvature at this point, q_0 [6], are given as

$$w_0 = \frac{\lambda}{\pi NA} \quad q_0 = i \frac{\pi w_0^2}{\lambda} = i \frac{\lambda}{\pi NA^2} \quad \text{Equation 7.}$$

We then use another matrix (not shown) with q_0 as the input beam curvature. This matrix travels from the focus point to the input of the beam splitter as shown in Figure 2. The output of this matrix calculation is q_{in} . The value for q_{in} is now a function of the NA, f , and d_t and is given as

$$q_{in} = d_t - f + i \frac{\pi NA^2 f^2}{\lambda} \quad \text{Equation 8.}$$

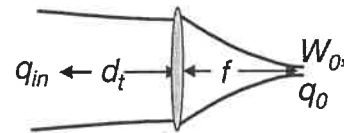


Figure 2: Schematic of Case 2, determining q_{in} .

The focal length, f , is calculated using the NA and the beam diameter. The calculated focal lengths for an input beam diameter of 8 mm and varying NAs are shown in Table 1. The NAs correspond to those of the microscope objectives used in our interferometer. The resultant q_{in} calculated using Equation 8 is also shown. The d_t parameter is adjusted based on the physical attributes of the micro-interferometer.

Table 1: The calculated f and q_{in} from the NA.

NA	f (mm)	q_{in} (mm) Case 2
0.28	13.7	$dt - 13.7 + i*73206$
0.42	8.6	$dt - 8.6 + i*65421$
0.55	6.1	$dt - 6.1 + i*55405$
0.70	4.1	$dt - 4.1 + i*40511$

5.3 The Remaining Inputs

We have shown two methods for determining the parameter q_{in} , as well as how to calculate the focal length based on the NA and the input beam size. The other relevant input parameters to be determined are d_t , d_r , and r . The distances in the test arm (d_t) and in the reference arm (d_r) are based on the range of the micro-interferometer, 50 mm to 100 mm and 150 mm to 300 mm, respectively. Finally r is defined by the test optics, and we are examining micro optics with radii ranging from 20 μm to 1 mm.

6.0 Obtaining Radius from the Model

Our goal is to determine the output radius of the test part and resultant error based on the input parameters described above. First, q_{in} and the focal length are calculated based on one of the methods described above. Next, q_{out} of the reference and test arms are calculated. A range of values are calculated for q_{out} by varying s , taking care to encompass the both the cat's eye and confocal positions ($f-2r < s < f+r$). The curvature, $curv$, for both the reference and test arms is defined as the real part of the inverse of q_{out} and is given as

$$curv = \text{Re} \left(\frac{1}{q_{out}} \right)$$

Equation 9.

Note that $curv_{ref}$ has a single value for all positions and $curv_{test}$ is a function of s , the distance between the lens and the test part.

The cat's eye and confocal positions occur when the $curv_{test}$ and $curv_{ref}$ match. To do this, we define $curv_{out}$ as $curv_{test} - curv_{ref}$. The cat's eye and confocal positions occur at the s position where $curv_{out}$ is zero. An example graph of the $curv_{out}$ function is shown in Figure 3 with the confocal and cat's eye shown. The intermediate curve match near 9.47 mm is the result of unequal beam radii for q_{in} and q_{out} and is easily discarded as a solution.

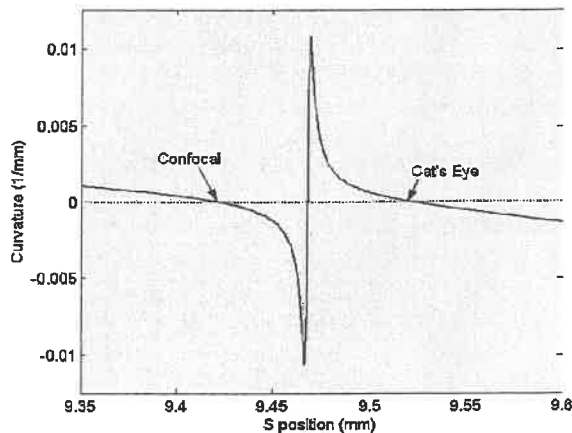


Figure 3: Example of a $curv_{out}$ function, $NA = 0.4$, $d_t = 100$ mm, $d_r = 150$ mm, $r = 100$ μ m, and $q_{in} = 90.48 + 71991i$ (Case 2).

The cat's eye and confocal positions were found by determining the s value where the function crossed the x-axis (in 0.1 nm sized steps) such that the position is certain to ± 0.1 nm. The radius

is then found by subtracting the confocal position, s_{CF} , from the cat's eye position, s_{CE} , and the radius error is computed:

$$Radius_{out} = s_{CE} - s_{CF}$$

$$Radius_{error} = Radius_{out} - r$$

Equation 10.

The error is positive when $Radius_{out} > r$, input radius, and negative if the $Radius_{out} < r$. The offset of the cat's eye position is determined by subtracting the focal length of the lens from the s_{CE} position and the confocal offset is then found from the radius error and the CE_{offset} :

$$CE_{offset} = s_{CE} - f$$

$$CF_{offset} = Radius_{error} - CE_{offset}$$

Equation 11.

By this convention, an offset toward the lens is negative and away is positive.

To summarize, the radius is found using the following steps:

- Calculate q_{in} and f
- Calculate $q_{t,out}$ (function of s) and $q_{r,out}$
- Calculate $curv_{out}$ as a function of s
- Determine the two s positions where $curv_{out}$ is zero
- Radius is the difference between the two positions

7.0 Results of the Code

We are interested in testing the error in the radius of the test part as it depends on several parameters. The nominal values for these parameters are based on the experimental setup. They are varied in simulation to study their effect on radius. The results of the simulations are given in the sections below. The conclusions based on these results are in Section 7.4.

7.1 Varying Radius

We first tested parts with varying radius using the two different cases to determine q_{in} . For all these cases $d_t = 50$ mm, $d_r = 150$ mm, $\lambda = 632.8$ nm, the index of the air is 1, the NA is 0.42, the aperture size is 8 mm diameter, and the focal length is 8.6 mm. We are testing radius ranging from 0.02 mm to 1 mm as shown in the graphs below. Figure 4 and Figure 5 show the radius error when using q_{in} from Case 1 and 2, respectively.

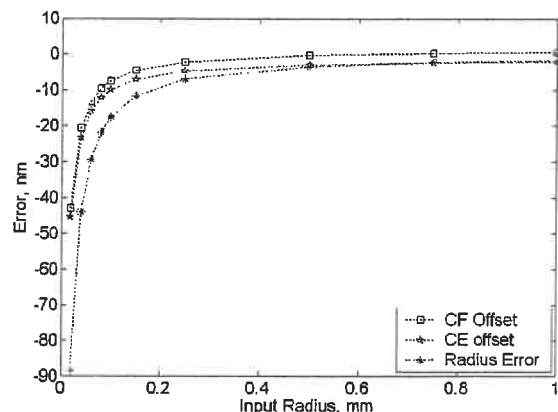


Figure 4: Radius Error for varying input radius using q_{in} from Case 1 ($q_{in} = 0 + 79433i$ mm).

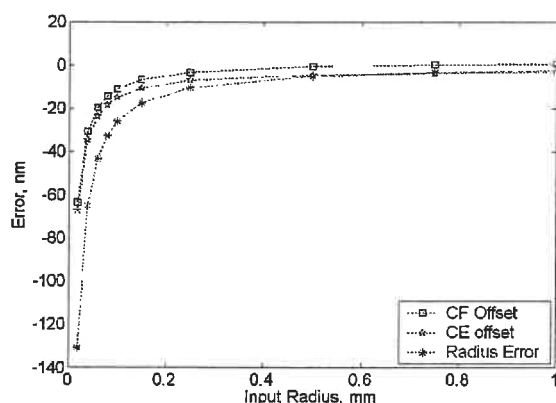


Figure 5: Radius Error for varying input radius, q_{in} from Case 3 ($q_{in} = 41.357 - 65421i$ mm).

7.2 Varying NA

Case 1, where q_{in} is calculated from the input beam diameter, is first considered. NA is varied, which affects the focal length of the lens. The input parameters are same as before and the NA ranges from 0.2 to 0.7. An input part radius of 500 μm was first tested and the results are shown in Figure 6.

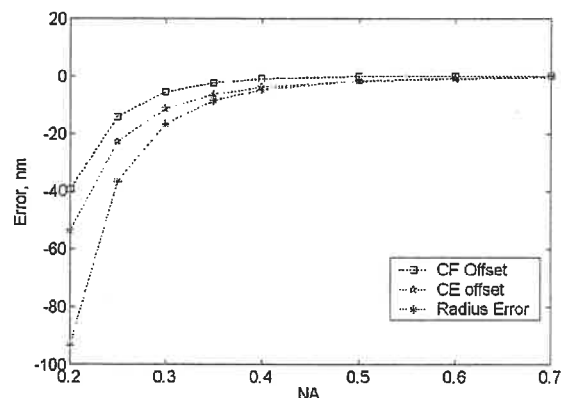


Figure 6: Radius Error for a 500 μm part, varying NA, q_{in} from Case 1.

Note that q_{in} does not change here; it is constant at $0 + 79433i$ mm, but the focal length does increase with decreased NA. Therefore, the radius error increases as focal length increases (decreasing NA). For a part radius of 250 μm , the trends are similar, but with a larger magnitude in error.

Determining the effect of NA on the part radius when using q_{in} from Case 2 is more complicated. This is because q_{in} is dependent on NA and focal length, which varies with NA. The distances d_t and d_r are the same as before and the input radius 500 μm . The results are shown in Figure 7.

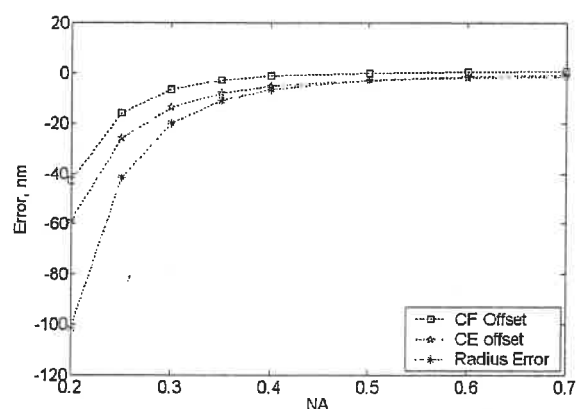


Figure 7: Radius Error in a 500 μm part, varying NA, q_{in} from Case 2.

This demonstrates the same trend as in Figure 6 and the errors are of the same magnitude, but the errors are not quite the same. This because the q_{in} changes with varying NA, unlike the previous case.

If the input radius is much smaller, 25 μm , the errors due to changing NA significantly increases to as much as 8% for a 0.2 NA.

7.3 Varying the propagation distances

The propagation distances, d_t and d_r , were varied and the resultant errors examined. The results (not shown) indicate that the magnitude of the radius error is similar to previous results, but does not vary with changing d_t or d_r . However, the cat's eye and confocal positions do vary, but, the difference between the two is always the same. This is true regardless of the method to calculate q_{in} . Therefore, the propagation distance is not a factor in the radius measurement, but may be a factor in other types of interferometric measurements. For example,

propagation distance would matter if the measurement is absolute and not differential.

6.4 Conclusions about the model

From the results above and others not shown the following observations are made:

- The measured part is always smaller than the input radius.
- CE and CF offset errors are on the same order of magnitude.
- Radius error increases (apparently exponentially) as the radius decreases.
- Radius error increases (apparently exponentially) as NA decreases.
- Propagation distances d_i and d_r do not affect the radius measurement

It is interesting to note that decreasing the NA (equivalent to stopping the beam down) will increase the errors in the radius. Yet, often a beam is stopped down to reduce spherical aberration and therefore reduce a bias in the measurement due to the aberration.

8.0 Planned Experiments

Experiments are planned to verify the conclusions above using the Micro-Optics Transmission and Reflection Interferometer (MORTI) at UNC Charlotte. This interferometer, built on a microscope body, is a standard Twyman-Green, $\lambda = 632.8$ nm, with an objective lens as its focusing lens. A laser scale is used to measure the distance between the cat's eye and confocal positions. The laser scale is calibrated using a displacement measuring interferometer, not an artifact as with other interferometers. Additionally, MORTI is located in a temperature controlled metrology laboratory.

A series of steel spheres will be measured on MORTI with the following planned experiments:

- Varying the radius of the test part
- Using different objectives (NA)
- Stopping down the input beam
- Moving the reference mirror (d_r)

9.0 Future Plans

Further tests to be evaluated using the Gaussian beam model introduced in the paper include introducing aberrations and the Zernike defocus term to the model. The traditional radius measurement uses the Zernike defocus term to define the cat's eye and confocal positions [6], but the Gaussian beam model uses the $curv_{out}$

function. We predict these two methods will yield the same result in a non-aberrated system. But, in an aberrated system (typically spherical) we predict the results will not be the same.

The relationship between the $curv_{out}$ function and the Zernike defocus term must be evaluated. An optical path difference map can be generated at each s position from the $curv_{out}$ function. Then the Zernike polynomial fit must be performed to find the defocus term.

Spherical aberration can easily be added to the model using propagation through a glass plate in the converging beam. Different thickness of glass plate (equivalent to varying the aberration) will be tested. In general, we predict that the radius error will increase with increased aberration.

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Interferometric Measurement of Sphere Diameter

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An interferometer-based system for measuring the diameter of spheres is being developed at University of North Carolina at Charlotte. Our goal is to measure spheres up to 3 inches in diameter with an uncertainty of 10 nm or less. To reduce sensitivity of the instrument to parameters like temperature, pressure and humidity, measurements will be performed in a controlled environment. To determine the size of the sphere under test, a number of corrections need to be applied to remove effects of deformation introduced by the measurement process itself. Based on the design presented in this paper, an error budget in accordance to 'Guide to the expression of Uncertainty in Measurement' has been developed.

Introduction

Measuring the absolute diameter of precision spheres remains a challenging task for metrologists. Previously, comparative techniques were considered sufficient for accurate quality control of the dimensional measurements. As the need for more accurate measurements evolves, almost all measurement systems utilize interferometry for calibration at the highest precisions. Gage blocks, representing a relatively simple geometry, are being calibrated all over the world using interferometry and today uncertainties in these measurements have been reduced to a few nanometers. Even though a sphere may be considered a relatively basic geometry, an instrument which can determine its diameter with the uncertainties comparable to gage blocks has yet to be demonstrated. In this current development, the same gage block measurement technique is adapted for measuring the diameter of spheres.

Precision spheres have lot of metrological applications including the ball bearing industry, Gravity Probe-B experiment [1], spindle error analyzers, Co-ordinate Measuring Machine references spheres and probe tips, Avogadro mass standard project [2] to name but a few. The motivation for development of this instrument is to make an optical measurement system with a lowest possible uncertainty. This work is based on an instrument at NIST which measures spheres.

State of the Art

Measuring spheres has its own importance and most national labs have their own dedicated sphere measurement facility. Measurement techniques vary from one laboratory to another. While most involve combinations of diameter and roundness measurements, it is clear that uncertainty in those measurements vary considerably.

Dimensional metrology facility at NIST (National Institute of Standards and Technology) provides calibration of typically high end precision spheres using roundness, surface form measurements and diameter measurements at multiple orientations. The Strang Monochromatic Fringe Viewer [3] at NIST is used to measure spheres up to one inch in diameter. It uses a

cadmium light source to illuminate the field of view. The concept behind the apparatus is forming a Fizeau Interferometer between two optical flats with the test sphere is placed between them. The method consists of measuring the distance between the two optical flats when the sphere is contacted with a defined force. The fringe fraction is calculated in a similar way as it is calculated for gage blocks [4], [5]. NIST uses the same apparatus mentioned in [3] for measuring spheres also (as of writing this has not been published). In all cases, measurements are performed in temperature controlled environments. The final value reported will be the undeformed diameter and an uncertainty budget associated with the measurement. The objective of our development is to expand the range of this apparatus for measuring spheres of up to 75 mm and also add software capability for computing fringe fraction at the center of the sphere.

Key Components of Metrology System

Key components of the measuring instrument are shown in figure 1 below. In this, the sphere will be contacted between two, parallel optical flats. Fizeau interferometry is used for obtaining the interference fringes between the two flats and between the individual flats and the sphere in a region surrounding the contact point. Precise adjustments are incorporated for aligning the two flats to obtain fringes.

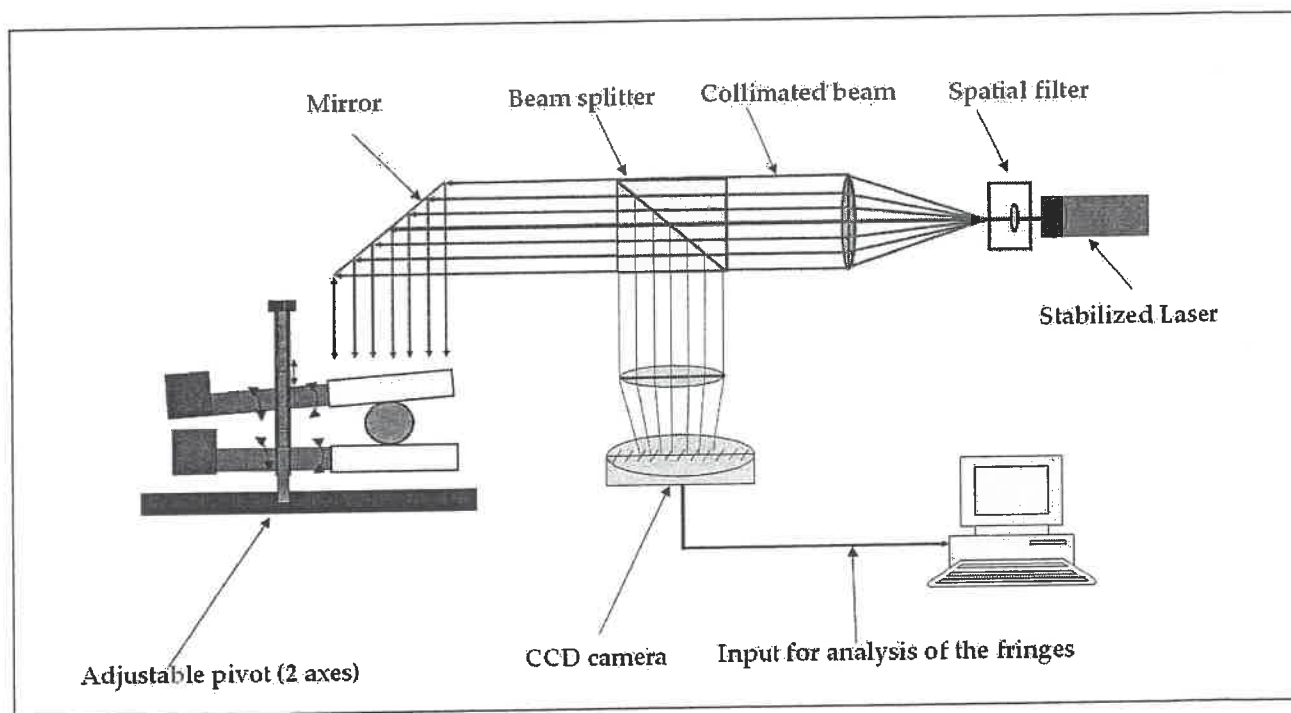


Fig 1: Setup of the Interferometer

The optical flats are housed in a 'see-saw' mechanism with a kinematic pivot and the contact force is carefully controlled using dead weights applied on the opposite side. To accommodate different diameters of sphere, the complete mechanism can be raised and lowered. Angular alignment for adjustment of the fringe spacing is achieved using fine adjustment screws, see fig. 2. In practice, the lower see-saw mechanism is below the top see-saw with parallel pivot axes. To

clarify understanding of the operating principle the lower see-saw has been rotated and shows the pivot axes to be perpendicular to each other in fig 2.

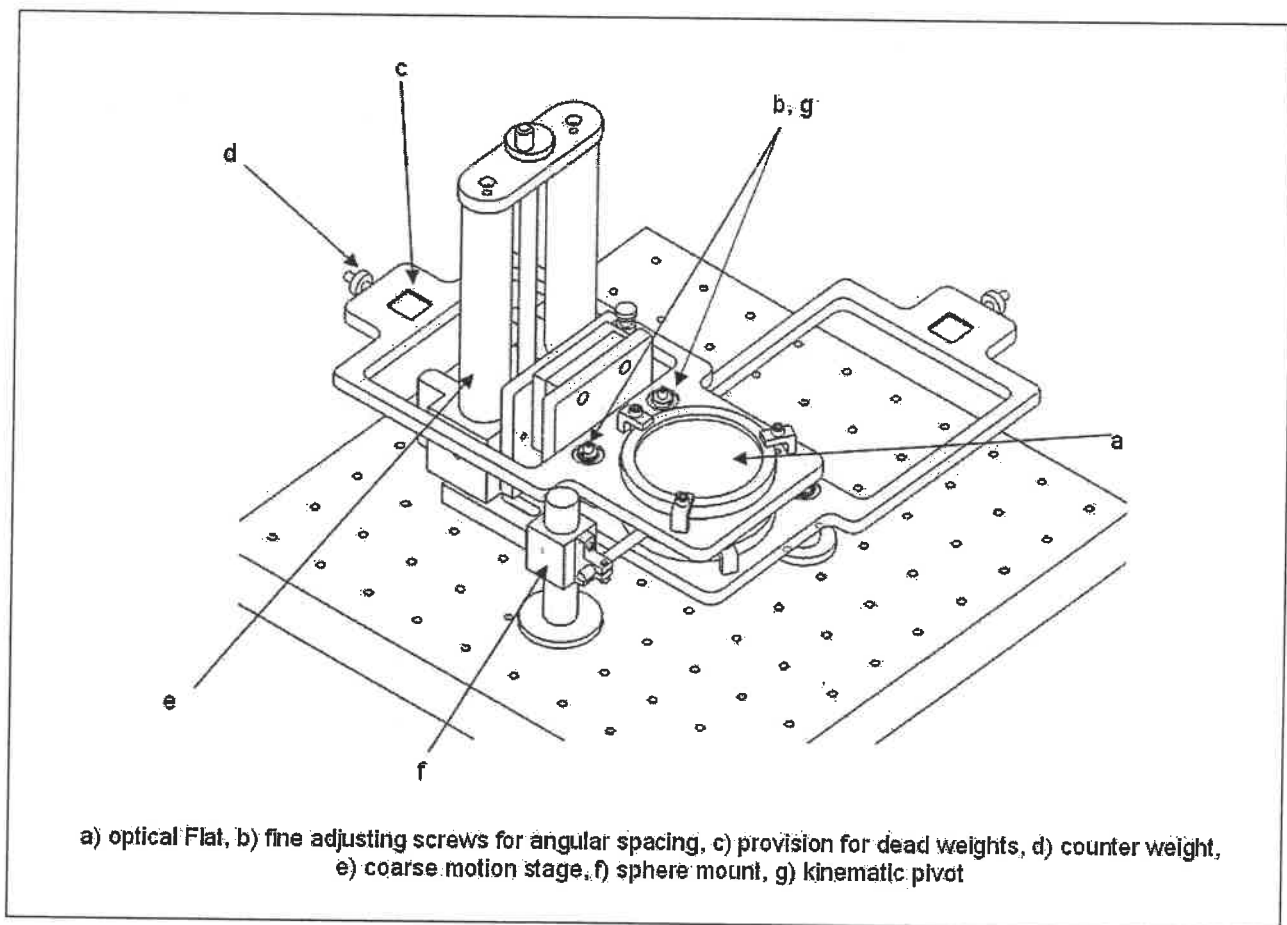


Fig 2: Solid model of the pivot mechanism

Desired forces are applied by adding or removing the weights while the test sphere is suspended on a fixed mount (f is fig 2). Both the top and the bottom optical flats are brought into contact with the sphere by adjusting the position of the dead weights relative to the 'see-saw' pivot.

By definition a perfect sphere has all the points on the surface at equal distance from the center. In practice, sphericity, surface finish/form, and out-of roundness constitutes some of the geometry related factors that deviates the sphere from its ideal condition. Three precision spheres of grade around 2.5 micro inches are being used for performing the experiments

Sphere is glued using a 5 minute epoxy to the end of a non magnetic steel shaft and held rigidly in this fixture (f in fig. 2). In these initial studies, to assess instrument repeatability, it is necessary to minimize diameter variations due to roundness errors by ensuring that measurements are taken at defined points across the spheres. To do this, the steel shaft can be rotated about its axis and has markings about its circumference that can be used to define the relative orientation at which different diameters are measured.

A stabilized He-Ne laser used to illuminate the field of view with the subsequent interference fringes being imaged using a CCD camera. Standard algorithms will be used to unwrap phase wherefrom fringe fraction is calculated. Algorithms will also be developed for providing a measure of the out of roundness of spheres and the separable errors in the optical flats. Multiple readings are taken in different orientations of the spheres to average out the errors due to these factors.

Round-robin tests are also being performed at NIST on the spheres used in this development are in progress and measurements obtained from this instrument will be compared.

Sources Contributing to Uncertainty

There are many factors that contribute to the uncertainty in the measurement. To get the undeformed size of the ball, a number of corrections need to be applied. Uncertainty is calculated in accordance to 'Guide to the expression of Uncertainty in Measurement'.

For high precision instruments and machines temperature is the major contributor for uncertainty. Temperature change especially in interferometry effects the measurements in two ways. Change in ambient temperature will change the refractive index of air and thereby changes the wavelength of laser. There will be change in the dimension of the part also. All these corrections can be added to the final value to get the undeformed diameter. However there is an uncertainty involved in temperature measurement itself which contributes to the final value. Assuming the Coefficient of thermal expansion of the material is known to 10% of its value. The standard uncertainty due to temperature is calculated to be $.08 \times 10^{-6} L$ (L is the diameter of sphere under test in meters).

Since the two optical flats are in contact with the sphere at two points. Hence there is Hertzian deformation [6] and has to be added to the final value. These equations hold well for smooth surfaces and the calculated standard uncertainty is $0.3 \times 10^{-9} \text{ m}$.

Wavelength corrections for temperature, pressure and humidity also need to be added. Edlen's equation is still widely being used for estimating these corrections. The difference between experimental measurements when compared to the Edlen's equation is insignificant [7], [8]. Standard uncertainty in the simplified Edlen's equation is 1.5×10^{-7} .

Optical flats used in this instrument are flat to 1/20 of the wavelength of laser being used. The optics used in the instrument also contains errors and have to be calibrated. The most common way to do it is by three-flat test. While it is known the flatness error can be substantially reduced [9], [10], an evaluation of the resultant uncertainty has not, as yet, been evaluated.

Acknowledgements

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Optoelectronic Digital Holographic Methodologies for Shape and Deformation Measurements

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Current demanding engineering analysis and design applications require effective experimental methodologies for characterization of surface shape and deformation of components. Such characterization is of primary importance in many applications since shape is directly related to the function of the components and changes in states of deformation to their accurate operation, performance, and integrity.

Recent advances in laser technology, optical sensing, and computer processing of data have led to the development of advanced optical metrology methodologies. These optical methodologies can provide noninvasive, remote, and full field of view information about the components of interest. Such information relates to changes in shape and/or size of the components, characterizes anomalies, and provides tools to enhance fabrication processes. In this paper, optoelectronic digital holographic (OEDH) methodologies that we are developing are described and their capabilities are illustrated with representative applications involving the testing and characterization of microelectromechanical systems (MEMS) [1].

MEMS are micron-sized electrical and mechanical devices fabricated using very large-scale integration (VLSI) techniques adapted from those of the microelectronics industry. MEMS define both, the fabrication process and the devices themselves, and represent one of today's most exciting areas of technological development activity. Recently, MEMS technologies have been developed and applied by a number of industries to produce components with unprecedented sensing and control accuracies and resolutions [2]. As the capabilities of MEMS become more widely recognized, however, it is also recognized that the biggest obstacle to growth of MEMS applications is the design cycle time, which depends on tightly coordinated application of advanced design, analysis, fabrication, and testing. Testing of MEMS involves measurements of their electrical, optical, and/or thermo-mechanical responses to the driving signals and/or environmental loading conditions. Furthermore, in order to understand mechanics of MEMS and materials used for their fabrication, advanced noninvasive testing methodologies, capable of measuring the shape and changes in states of deformation of MEMS packages and materials subjected to actual operating conditions, are required.

There are a number of quantitative methodologies being used to perform testing of MEMS. However, methodologies are either very costly, time-consuming (e.g., use of contact and/or scanning modes), or are not capable of providing the types of measurements that are necessary for the effective testing and characterization of MEMS packages.

This paper will address some of the challenges imposed by the need for super-high resolution measurements, including testing components with

- geometrical discontinuities (e.g., capacitive pickup mechanisms and devices with a high-aspect ratio, including integrated electromechanical components and IC circuits), and
- enabled with mechanisms subjected to 3D motions and high-frequencies of operation (electromechanical components of MEMS can reach very high oscillation frequencies, i.e., larger than a few-hundredths MHz and future requirements call for a few GHz).

Other challenges include:

- the full-field of view measurement capabilities with sub-nanometer measuring accuracy;
- the measurement of a large number of components, e.g., testing at the wafer level;
- the testing at the different levels of the electronic packaging and under realistic operating and loading conditions; and
- the compatibility of the output data with CAD environments that allow the definition of models and conditions suitable for multiphysics computational analyses.

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SOPHISTICATED LIGHT SWITCHES

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If optical metrology systems reach their limits in terms of dynamic range or speed, the use of sophisticated light switches might be a way to bring more flexibility into the measurement process. Rather than just describing the action of turning on and off light, the term “switching” refers to more advanced adaptations of light such as spatially resolved intensity modulation, phase shifting, beam steering or wavefront shaping. Control over properties of light is commonly obtained using switches, modulators or spatial light modulators (SLMs). Many devices have become available on the market, with the number of addressable parameters ranging between one and several million.

The paper gives an overview of the broad field of available light modulators in section 1. Sections 2 - 4 provide an insight into a few interesting applications in which high resolution liquid crystal displays (LCDs) are used to create highly flexible tools. A phase shifting point source array and its application in an interferometer is described in section 2. Section 3 deals with a Shack-Hartmann sensor that uses adaptable microlenses. Section 4 describes the holographic optical tweezer, a flexible tool used to manipulate particles in the micrometer range.

1. Commonly used light modulators

The sensing and the control of light are the two main tasks in optical metrology. There are cases in which the mere sensing of light with a constant light source provides enough information. Often, however, a modulation of the used light provides much more information so that in the subsequent processing of the gained information the measurement result can be calculated. Figure 1 shows a brief overview of commonly used light modulators. These are divided into modulators that influence the intensity, the phase or the wavefront shape of the involved light. Another distinction can be made by looking at the number of parameters that can be influenced with the modulator. In the figure, the number of parameters is indicated by the size of the used font.

The simplest thinkable modulation of light is the switching of light sources by means of simple electrical switches, or mechanical shutters. This binary intensity modulation can be applied for calibration purposes or illumination sequences from different angles. With electronic controllers for diode lasers in fiber telecommunication the switching process is taken to extremes. Acousto-optical modulators (AOMs) are popular for fast modulation of light beams. A piezoelectric transducer is used to excite a sound wave in a transparent medium. The sound wave produces a periodic spatial modulation variation in the medium which acts as a volumetric phase grating. This grating can efficiently diffract the beam into one of the grating's diffraction orders. By varying the amplitude of the acoustic wave the diffraction efficiency is changed. The second useable parameter is the frequency (period) of the acoustic wave and thus the diffraction angle (tilt of the wavefront) of the diffracted beam. In some applications the frequency shift introduced by interaction with the moving wave is also used [1].

Piezo translators (PZTs) are part of most phase-shifting interferometers. They introduce a pure phase shift by moving the reference mirror of the interferometer by fractions of the wavelength. Some telescopes use a segmented mirror array with up to several hundred PZTs, so that static and dynamic phase aberrations can be removed. Another phase shifting device is the liquid crystal phase shifter (LCPS). The phase shift is introduced by changing the orientation of liquid crystal molecules in a thin cell by applying an electric field. Depending on the polarization of the incoming light with respect to the orientation of the molecules, the device can also change the polarization state of the light. Typical applications for such LCPSs are in the fields of phase-shifting polarization interferometry and phase-contrast microscopy.

The top set in figure 1 represents the class of modulators that influence the shape of the interacting wavefront. In the simplest case, a tilt is introduced to the wavefront by flipping solid mirrors about one or two axes. This is achieved by galvano-scanners. Other devices enable the control of optical power without any moving parts. Liquid crystal lenses have a laterally varying phase shift realized either with a convex or concave liquid crystal cell or with a high resistivity electrode causing a weaker electric field in the center of the lens than at its edge. Electrowetting lenses [2] operate upon the effect obtained when the contact angle of a liquid/liquid-interface with a solid casing changes as voltage is applied to the casing. Due to the fact that the two liquids have different indices of refraction, the modified shape (curvature) of the interface causes a change in the optical power of the lens. An application of these lenses is expected to be used in the next generation of mobile camera phones. Deformable membrane mirrors (DMMs) offer more degrees of freedom. They usually consist of a metallic membrane with up to nearly 100 underlying electrodes. Voltages on the electrodes generate electrostatic forces which determine the shape of the mirror. Due to their fast response time, they are often used in compensation optics for the removal of the influence of atmospheric turbulence in ground-based telescopes. Other types of membrane mirrors use thermal expansion, magnetic forces or piezoelectric actuators for the membrane deformation. The most variability is offered by spatial light modulators (SLMs) such as liquid crystal displays (LCDs) or micro mirror arrays (MMAs), basically consisting of arrays of up to millions of liquid crystal cells or movable mirrors which can be individually controlled. Operation modes vary between binary amplitude modulation (flip MMA, ferroelectric LCD with polarizers), continuous amplitude modulation (LCD with polarizers), binary phase modulation (ferroelectric LCD) and continuous phase modulation (piston MMA [3], LCD). Wavefront shaping is best achieved with the continuous phase modulators. Wavefront deviations of many wavelengths, however, can be achieved in a diffractive manner with all 4 types, whereas the best efficiencies are achieved with continuous phase modulation capability between 0 and 2π . The devices then act as flexible diffraction gratings where the local grating period determines the local deflection angle. One disadvantage of pixelated devices is the appearance of unwanted diffraction orders. It is advisable to characterize the device prior to its application [4].

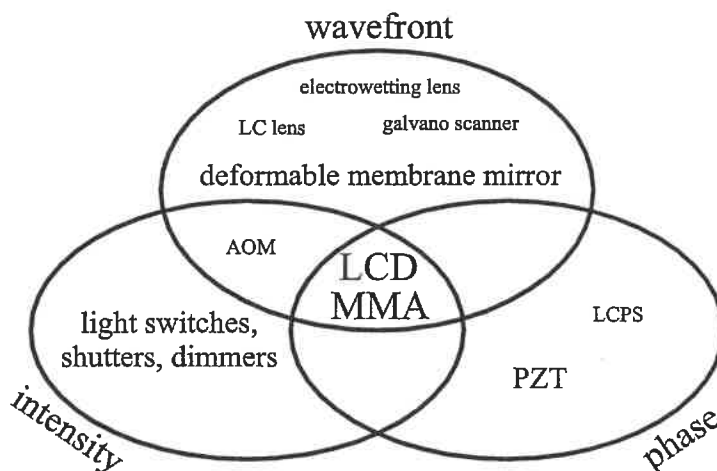


Figure 1: Overview of the most commonly used light modulators showing devices that modulate intensity phase and wavefronts. The font size indicates the number of controllable parameters (smallest for 1 or 2, intermediate for 10 to 1000, and largest for up to millions)

SOPHISTICATED LIGHT SWITCHES

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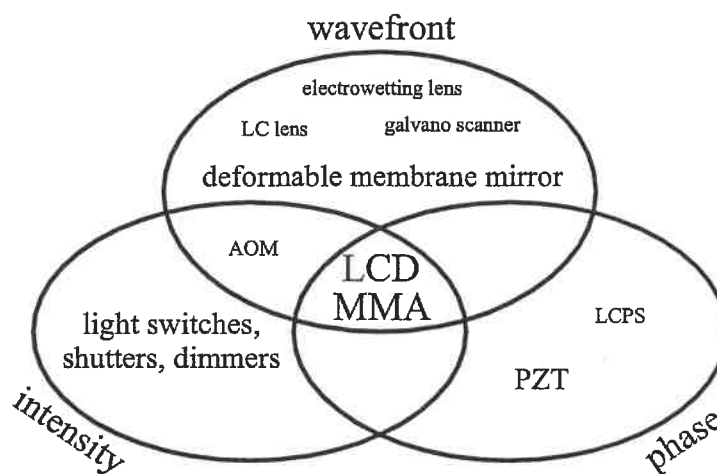


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2. Phase shifting point source array

The phase-shifting point source array (fig. 2 left) is a device that enables free intensity regulation and independent phase shifting of point sources arranged on a regular grid. It consists of a high-resolution liquid crystal display (LCD) followed by a microlens array and an array of pinholes aligned centrally behind the microlenses at a distance that corresponds to the focal length of the microlenses. The pinholes constitute the potential positions of light sources. The illumination of the PPA is monochromatic, collimated and is aligned with a slight tilt. The LCD pixels are electrically addressed and controlled by software to be either transparent or non-transparent (in principle, any modulation would be suitable, e.g. also phase modulation). When the entire LCD is non-transparent, all point sources are inactive. Even if part of the light is transmitted by the LCD (contrast ratio of LCDs typically 1:250), it will be blocked since the tilted light is focused onto a spot next to the pinhole. In order to activate a point source, a local linear grating needs to be displayed by the LCD in front of a microlens. The grating causes several diffraction orders which generate a line of focal spots in the plane of the pinholes. The tilt of the PPA illumination is aligned in such a way that only the first diffraction order goes through the pinhole. The point source is now active. This arrangement also allows a phase-shift of the light emitted by the point source by laterally shifting the displayed grating. The amount of phase shift is calculated using the following equation:

$$\Delta\varphi = 2\pi \frac{d}{P},$$

where d is the lateral shift and P is the period of the displayed grating. The intensity of the light source can be adjusted by displaying gratings with varying diffraction efficiencies.

The interferometer shown in fig. 2 (right) was equipped with a PPA. Together with the collimation lens, the PPA generates reference waves with varying tilt angles and also provides the phase shift for standard phase shifting interferometry algorithms. In this way a common problem in aspheric testing - the inability to evaluate regions with too high fringe densities - could be greatly reduced since the fringe density depends on the angle between object wave (the one reflected by the surface under test) and reference wave. Several measurements with different reference wave tilts are combined to reveal the overall surface shape. Due to the fact that no mechanical actuators are used in this setup, the results are highly reproducible. Furthermore, the different reference waves can be calibrated to each other by switching on two references waves at once and phase-shifting one of them while the object wave is blocked. More details about the method are given in [5].

The fields of coherent fringe projection or digital holography offer other possible applications for the phase-shifting point source array.

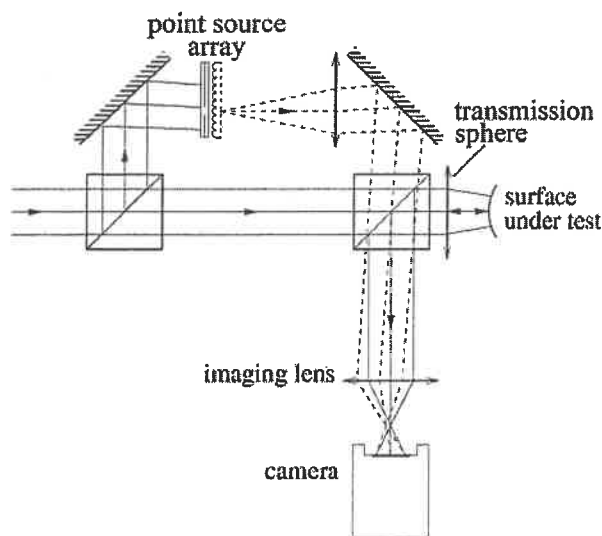
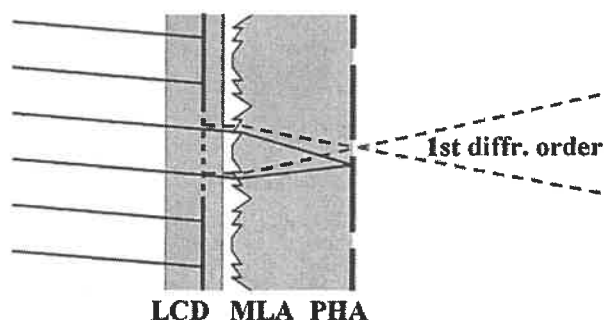


Figure 2:

Left: View of a section of the phase-shifting point source array consisting of a liquid crystal display (LCD) a microlens array (MLA) and a pinhole array (PHA).

Right: interferometric setup with the incorporated phase-shifting point source array.

3. Adaptive Shack-Hartmann sensor

A Shack-Hartmann sensor (SHS) is a wavefront sensor used in a wide range of applications such as adaptive optics in astronomy, laser beam analysers and non contact measurements.

In a conventional SHS, a static microlens array is used. The wavefront is sampled by the microlens apertures and brought to a focus on a camera chip. The focus positions are measured and compared to the focus positions measured with a plane reference wavefront. The displacements are proportional to the local wavefront tilts which are used to calculate the wavefront shape.

In the adaptive SHS [6] the static microlens array is replaced by a spatial light modulator that displays freely programmable diffractive microlenses. In our experimental setup we use a high resolution liquid crystal display (LCD, 1024x768 pixels) in amplitude modulation. In the basic setup each microlens is a Fresnel zone plate. Sensor parameters such as focal length, aperture size and number of microlenses can be modified according to the measurement task. A long focal length of the microlenses, for example, favours measurement precision but limits the maximal measurable wavefront slope.

The measurement dynamic of the conventional sensor is limited by the fact that spots leave their sub-apertures (defined by the center of the microlens and its size) when large wavefront slopes are present (fig. 3). An unambiguous assignment of the spots to their microlens is not possible. There are special spot search routines that try to solve this problem. The adaptive sensor solves this problem by switching on or off single lenses or groups of lenses in order to identify the corresponding spots. The measurement dynamic is thereby increased.

Precise measurements require a precise determination of focal spot positions. This is not always possible because curvatures in the incoming wavefront cause blurred focal spots. With the adaptive sensor these curvatures can be considered in the design of each microlens. However, an approximate knowledge of the wavefront shape is necessary. This knowledge can be gained by making a preliminary measurement with uncorrected lenses. The adapted microlens design results in a point-like focal spot (see fig. 3). Therefore the measurement accuracy is increased.

Very strong curvatures in the wavefront could result in an overlap of neighbouring focal spots. The adaptive sensor can overcome this problem by adding a well-defined tilt to the microlens hologram. This increases the dynamic range.

The intensity of focal spots can strongly differ due to an inhomogeneous illumination. This decreases the signal-to-noise ratio for dark spots. The intensities of the spots can be balanced by adjusting the efficiency of the diffractive microlenses. The camera can then be used with the maximal possible signal-to-noise ratio.

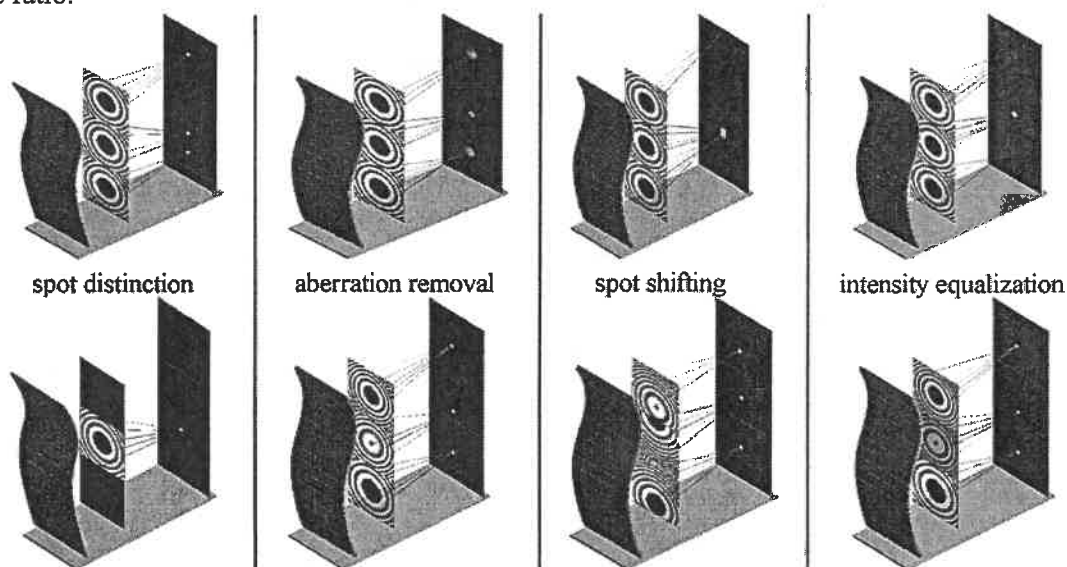


Figure 3: Features of the adaptive SHS.

4. Holographic optical tweezer

Optical tweezers use a focused beam of light to trap particles which have a size ranging from a few hundred nanometres to several tens of micrometers. Due to the non-contact character of optical tweezers they are very suitable for handling microbiological objects. Moving the trapped particles is possible by steering the focussed beam by means of mechanical actuators such as tiltable mirrors and movable lenses. Multiple traps can be established with beam splitters and the same number of mechanical actuators for each optical path. However, systems become quite complex if three or more traps are desired. The use of two acousto-optical modulators (AOMs) is one way of generating multiple traps [7]. The deflection angle of the (AOMs) is quickly switched from one trap to next, so that for each trap a pulsed optical trap occurs. Our approach, the holographic optical tweezer (see fig. 4), features even more flexibility. We incorporate a spatial light modulator (SLM) such as a micro-mirror array (MMA) or a liquid crystal display (LCD). Like optical tweezers with AOMs, holographic optical tweezers offer the ability to move many particles independently of each other without using mechanically moving parts. Furthermore, they allow us to move the particles also along the optical axis. The steering of the focal spots is done by placing the SLM in the Fourier plane of the focal spot and displaying "holograms" such as linear blazed phase gratings or Fresnel zone plates for the lateral or the axial motion respectively. A combination of both allows a motion in any direction in 3D space. Multiple traps are generated by displaying holograms that are calculated using a complex addition of the single trap holograms. The SLM can also be used to remove aberrations in the optical system so that a small focal spot is guaranteed [8]. Likewise, an artificial spot deformation is possible by modifying the single trap's hologram. Adding astigmatism, for example, generates a focal line. Adding a phase spiral results in a doughnut-shaped spot (intensity ring) which shows an increased trapping efficiency in some cases and can be advantageous when used with biological objects with light sensitive parts in the center.

The current work at the Institut für Technische Optik concentrates on force measurements and using the graphical processing unit (GPU) of the graphics board for rapid hologram calculations.

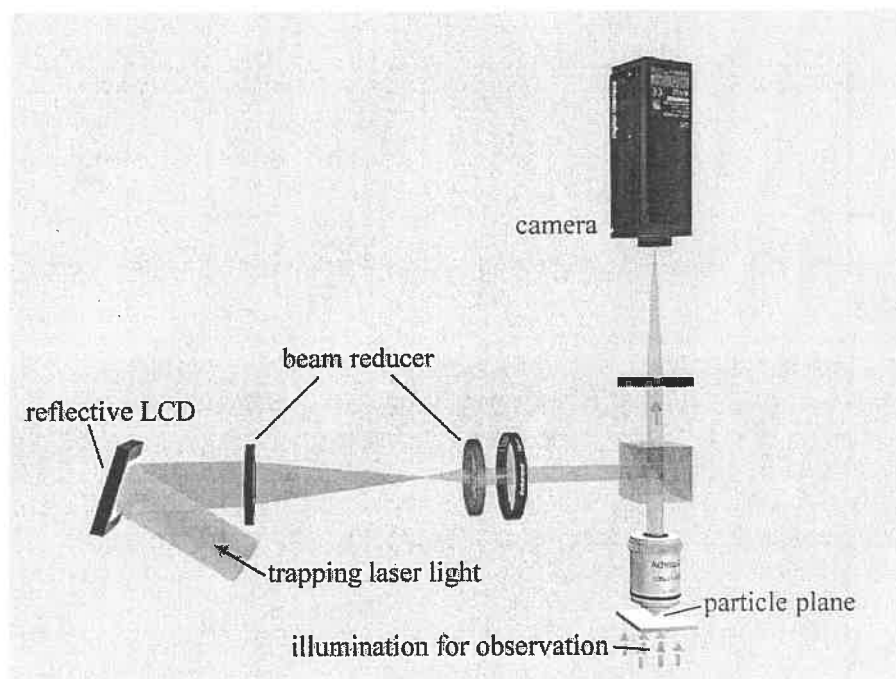


Figure 4: Sketch of the holographic optical tweezer setup. Particles are trapped and manipulated in the particle plane. All functionality is achieved by displaying holograms with the LCD.

5. Summary and Conclusion

As the three examples have shown, light modulators, especially the high resolution spatial light modulators with millions of individually-controllable pixels such as LCDs or MMAs, offer the possibility of bringing more flexibility into a great range of applications. Though some of the devices modulate only the amplitude or the phase up to a maximum phase shift of about one wavelength or less, complex phase functions with up to several hundred wavelengths of phase shift can be achieved when the devices are used as flexible diffraction gratings. A fact that must be considered is that the periodic pixel structure of the elements generates unwanted diffraction orders which also limit the amount of usable light.

Acknowledgements

We thank the BMBF (Federal Ministry of Education and Research), the Landesstiftung Baden-Württemberg and the DFG (German Research Foundation) for the financial support.

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Tomographic methods to characterize three-dimensional refractive index inhomogeneities

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Abstract:

For high precision optics the imaging quality depends among other factors on the three-dimensional distribution of the refractive index. To increase the performance of such systems the inhomogeneity of the glass blanks should be characterized. Optical phase tomography is a method to characterize three-dimensional refractive index inhomogeneities. We present some promising results for isotropic inhomogeneity and anisotropic inhomogeneity.

Keywords: tomography, glass, homogeneity, refractive index, volume interferometry

1. Introduction

In optical engineering of high precision optics glass homogeneity is a common problem. There are different methods to measure the homogeneity with an interferometric accuracy [1, 2, 3, 4, 5]. Optical glass has a refractive index variation in the range of 10^{-5} to 10^{-6} and smaller. This is usually given as a statistical quality measure. However for optical design the volume homogeneity is relevant as is illustrated in Fig. 1.

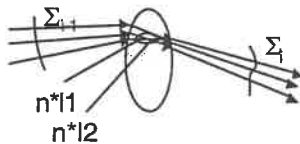


Fig. 1. Influence of volume inhomogeneities to optical image formation.

Inhomogeneities in glass blanks lead to ray displacements and phase retardations. Modelling the refractive index variation with a coplanar plate, as indicated in Fig. 2., shows that variations of the order of 10^{-6} can cause displacements as well as phase retardations of up to 100 nm. If the inhomogeneity is close to the image field the ray displacement is dominant and in the vicinity of the pupil plane the phase retardation is relevant.

For optical systems with high performance requirements both deviations are fatal. However the homogeneity can be included into the design process if a 3D model of the refractive index distribution is available. With such a model rays could be traced along their actual path and an individual optimisation could be used to enhance the system performance.

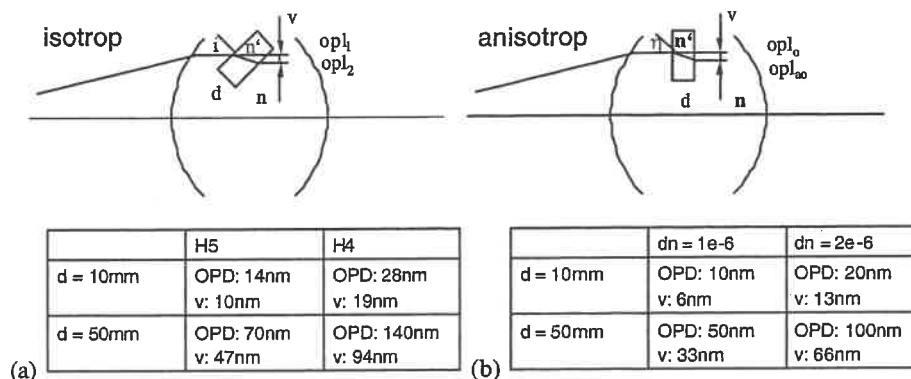


Fig 2: Model to estimate the influence of inhomogeneities in (a) the isotropic case (b) the anisotropic case

If polarisation is also considered the more general model of tensorial refractive indices must be used. Due to production processes and stress conditions the anisotropic inhomogeneity is of the same order as scalar inhomogeneity. For high-resolution projection optics a residual birefringency less than 2nm/cm lead to a performance degeneration of about 5% [6]. With a tensorial treatment and a Jones raytracing procedure inhomogeneous anisotropic media can also be included into the design process [7].

2. Reconstruction Procedure

In classical tomography many algorithms are available [8]. The basic idea of tomography is the Fourier slice theorem (Fig. 3.). The Fourier transform of a projection measurement is equivalent to one slice of information in the Fourier domain inclined in the projection angle. An individual ray corresponds therefore to a single point in the Fourier domain. The numerical challenge is to interpolate the data appropriately.

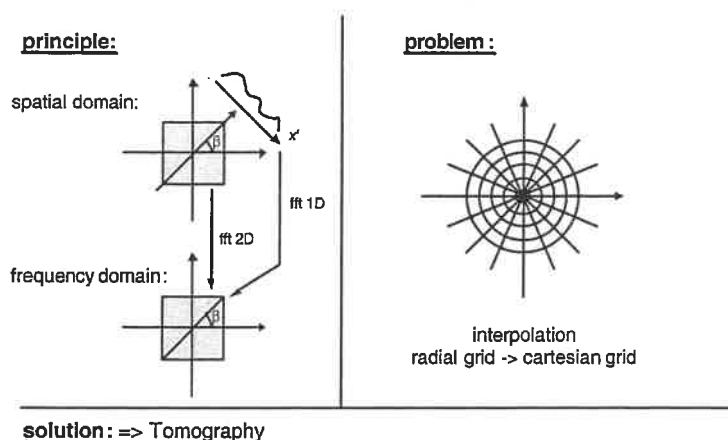


Fig. 3. Fourier slice theorem, the basic idea of tomography

For the adaptation to optical phase tomography we have developed an iterative method based on the algebraic reconstruction technique. A flow diagram is shown in Fig. 4. The volume under investigation is divided into discrete cells. Within these cells the optical properties are assumed to be constant. To compute the integral effects of inhomogeneities a ray tracing procedure is performed. This connects the measured values with the cells and allows the use of the weighted distributed back projection. Since the expected inhomogeneities are small in magnitude refraction, diffraction and beam splitting in the volume were neglected.

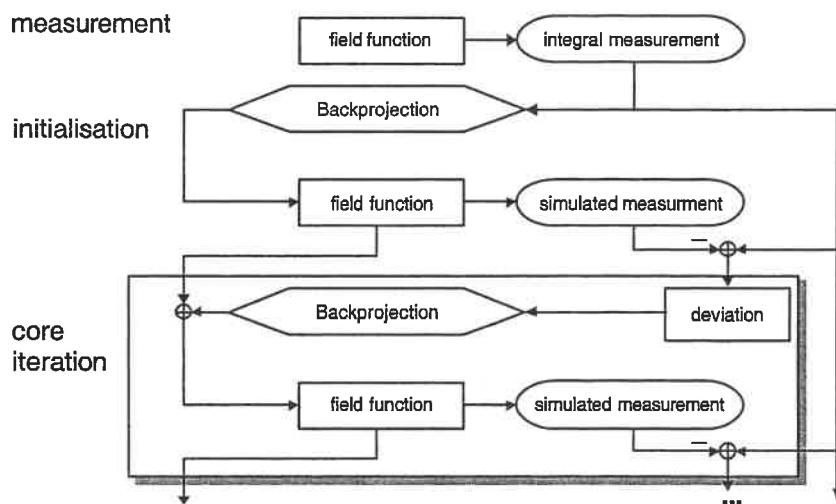


Fig. 4. Flow diagram of the optical phase tomography

3. Results for the isotropic inhomogeneity

The optical phase tomography in the isotropic case takes interferometric phase information from several projection measurements and reconstructs the refractive index volume information. Due to total internal reflexion not all the half space is accessible. In a disc like glass blank the reconstructable frequencies are inside a cone with the tip in the centre as shown in Fig 5. This leads to reconstruction artefacts.

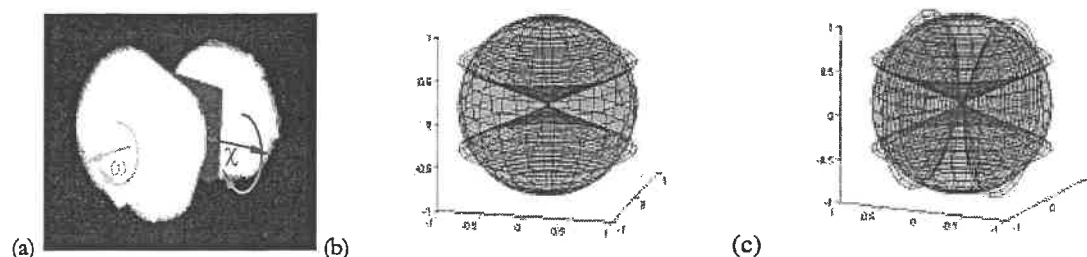


Fig. 5. (a) Model of a disc like glass blank penetrated by physically accessible rays. (b) Frequency domain of an arrangement with one window. (c) Frequency domain of an arrangement with two windows.

A flat simulation is shown in Fig. 6. to demonstrate the effect. It is assumed that there are two windows, one from -30° to 30° and a second ranging from 60° to 120° . Reconstructing from the first window results in a higher contrast perpendicular to the principle direction. Features along the principle direction are hard to reconstruct. However the second window gives access to this direction. Due to the linearity of the radon transform and its inverse the overall result can simply be averaged.

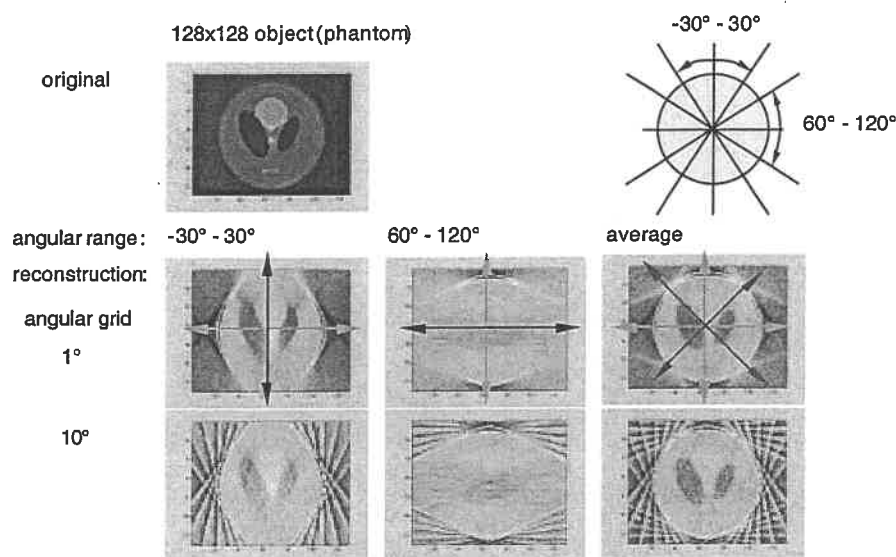


Fig. 6. Artefacts in reconstructions with principle directions.

A second interesting question in optical phase tomography from an engineering point of view would be: Is there an optimum between spatial resolution and metrology effort? See therefore Fig. 7. The different columns have different resolutions and the two rows have different metrology effort: 1° angular projection grid and 10° angular projection grid. For any reconstruction the detector grid, is automatically [8] chosen to an optimum and is indicated by n. For the 1° grid the resolution of 256×256 has the optimum contrast. With the 10° grid the high resolution result is reduced in contrast due to artefacts. Lower reconstruction resolution reduces the maximum reconstructable feature size.

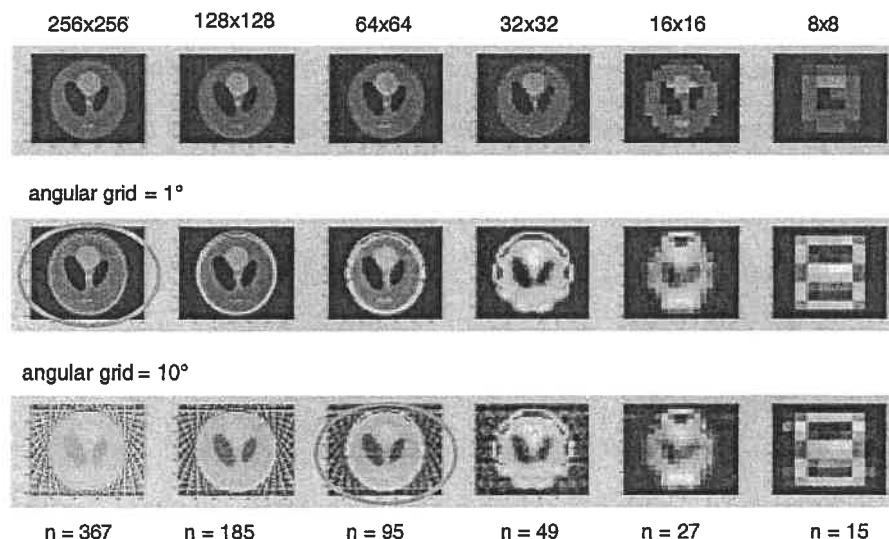


Fig. 7. Original (first row), reconstruction 1° angular projection grid (second row), reconstruction 10° angular projection grid (third row)

As a feasibility example for a real glass blank fig. 8 has been calculated. It shows the reconstruction with 10 projection directions of 3 different angles $\chi = 0^\circ, 15^\circ, 30^\circ$ and the azimuthal angles in the scheme. The correlation between the given isotropic inhomogeneity and the reconstruction was 0.953 on a 36 by 36 by 6 volume grid with a total of 10302 rays being used.

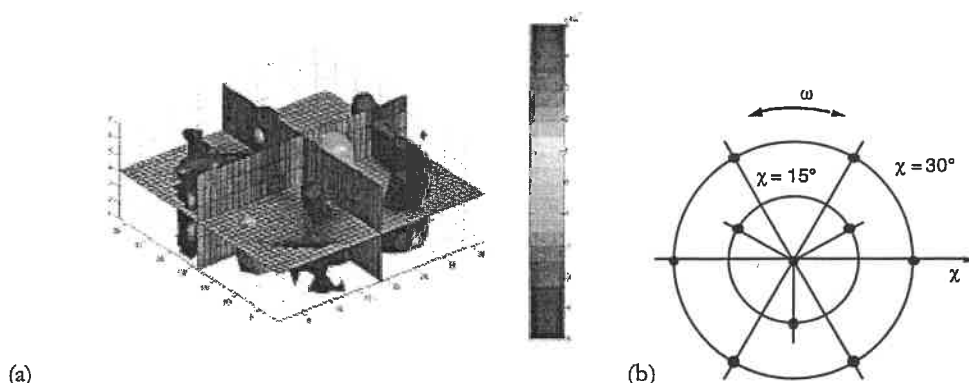


Fig. 8. (a) Reconstruction of an isotropic inhomogeneous glass blank. (b) Scheme of the projection arrangement

4. Results for the anisotropic inhomogeneity

The iterative tomographic reconstruction technique described above can be adapted to anisotropic volume inhomogeneities [8, 9]. This tensor tomographic method reconstructs 3D tensor fields of glass blanks out of Jones matrix measurements taken from different projection directions. The 3D tensor fields are the anisotropic refractive index tensor variation. Fig. 9 shows a simulation of such a reconstruction. Given was a random distribution of the refractive index tensor elements. Each tensor element is displayed in an individual slice plot so that nine diagrams make up the anisotropic inhomogeneity of a glass blank. With 10 projection directions and 17 by 17 measurements per direction a cube of 15 by 15 by 15 cells was reconstructed. After 1000 iteration steps the relative reconstruction deviation was smaller than 7.5%. Further investigations has shown that even with noise affected measurement data, with about 10% noise, the reconstruction succeeds.

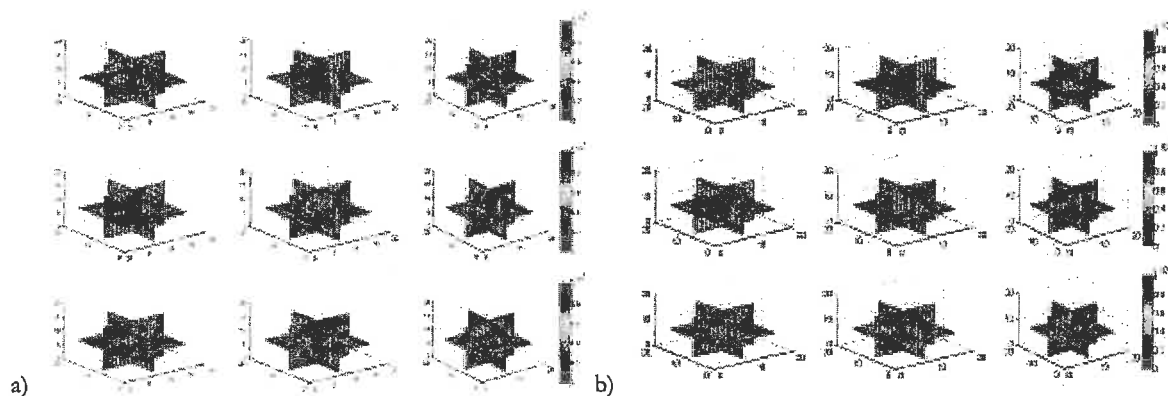


Fig. 9: Anisotropic inhomogeneity of a glass blank a) original b) reconstruction

A first measurement on a fused silica cube (18cm x 18cm x 18cm) is shown in fig. 10 (a) and (b). Just three projections from the sides of the cube were used, with 15 x 15 measuring points. The Jones transition matrices were measured with a commercial Jones polarimeter at Carl Zeiss SMT AG. The reconstruction to a 17x17x17 cell model is shown in fig. 10 (c). The diagonal tensor elements can be resolved in depth. However due to the tensor projection the off diagonal elements cannot be resolved in depth with just three projections. Also on the cubes the effect of corner clamping and gravitation effects on birefringence can be seen. The consistency of the reconstruction was tested by simulating a measurement with the reconstructed anisotropic inhomogeneity. Comparing that to the actually measured values results in a correlation better then 0.95.

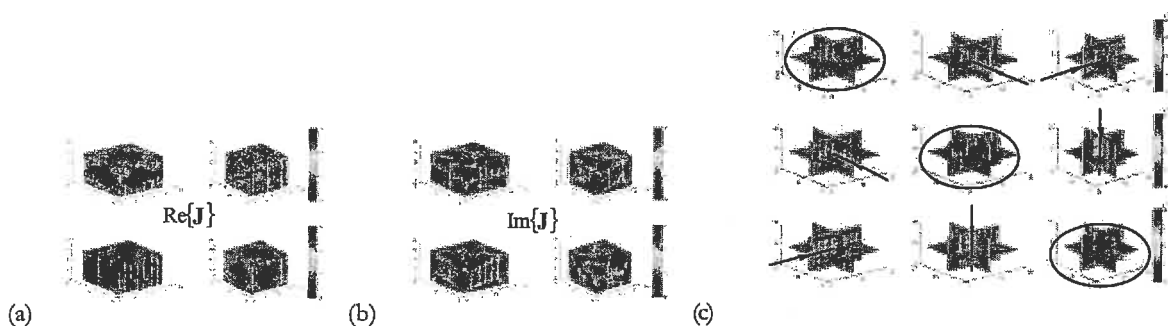


Fig. 10. (a) Real and (b) imaginary part of the measured Jones matrix from a fused silica cube from three orthogonal projection directions. (c) Reconstruction to 17x17x17 cells

5. Conclusions

Isotropic as well as anisotropic volume inhomogeneity is an important issue in high precision optics. The imaging quality degenerates since the glass which is available is not 100% homogenous and also clamping and gravitation effects are not zero. However the results of optical phase tomography showed promising reconstructions for the characterization of the isotropic and anisotropic inhomogeneities.

6. Acknowledgement

The contributions from Michael Totzeck, Birgit Enkisch, Ralf Müller and Daniel Krähmer and the financial support by the Photonics BW are gratefully acknowledge.

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Interferometric Metrology for Multi-beam Displacement Measuring Interferometer Assemblies

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Multi-axis displacement measuring interferometer assemblies present two distinct advantages to the integrators of closed-loop stage metrology systems.

1. Much of the system alignment task is transferred to the interferometer manufacturer.
2. It enables use of designs which are inherently more compact and stable.

In this paper we describe interferometric metrology techniques applied to multi-axis, two-pass plane mirror type interferometers for use in heterodyne motion metrology systems. These measurement techniques facilitate the adjustment and confirmation of:

- Axis position relative to mechanical datums
- Axis parallelism
- Axis signal efficiency

These parameters can be evaluated through non-interferometric techniques, though as specifications become more rigorous, interferometric techniques are advantageous.

Several of the parameters of interest are not easily measured. These are derived from other measurements of the multi-axis interferometer assembly. Measurements made include:

- Primary and secondary beam positions relative to mechanical datums
- Primary and secondary beam pointing
- Systematic errors of the measurement system to apply as corrections

In-process metrology was based on a phase-measuring, Fizeau interferometer. The Fizeau interferometer was illuminated via an optical fiber from an external laser. A portion of the light from the laser was split off and used to coherently illuminate the interferometer assembly under test. The measurement geometry provided for interferometric measurement of the primary and secondary beams of the interferometer under test, as seen from the target mirror. The Fizeau interferometer system collected both phase data and modulation depth data. The former was used for beam pointing and wavefront quality measurement. The latter was used for beam position measurement.

The measurement setup, measurements techniques, and analysis were successfully used to align and qualify multi-beam interferometers. Accuracy of measurement, and post-correction algorithms, relied on the use of several optical and mechanical references. Axis position accuracy was established to the level of 25 microns. This easily met the requirements of the project. Beam parallelism accuracy was at the level 10 micro-radians. Corrective improvements for this measurement will be discussed.

SUPER OBLIQUE INCIDENCE INTERFEROMETER USING SWS PRISM

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Abstract

A super oblique incidence interferometer is proposed by using an anti-reflection prism with sub-wavelength structure (SWS). Its sensitivity is lower than an ordinary interferometer with high contrast. A fringe interval of its interferometer depends on the incident angle. The SWS prism has a feature of increasing transmittance even if near the critical angle. The rigorous coupled wave analysis (RCWA) is used to analyze the anti-reflection effects of SWS. An experimental result of transmittance of SWS prism agrees well with the calculated result. High contrast fringe is achieved at 86° of the incident angle. A flatness measurement of a silicon wafer with circuit patterns is succeeded by this interferometer.

Keywords oblique incidence interferometer, flatness measurement, SWS(sub-wavelength structure), anti-reflection prism

1 Introduction

A surface profile measurement of silicon wafer is still important in the field of the semiconductor industry. A size of wafer is larger than 300 mm of the diameter and its thickness is thinner and thinner. Moreover, a fabrication process is more complicate for multilayer interconnection of the circuit patterns. Therefore, the flatness and/or waviness in this large area become bigger and bigger. Meanwhile, the requirement of the flatness in the micro area is more accurate using a process of the chemical mechanical polishing. There are many reports to overcome contrariety requirements¹⁾. The fringe interval of a common interferometer with vertical incidence is half of wavelength. If the flatness is much waviness, we can not detect the surface profile because of 2π uncertainty and/or overlapped fringes. In general, an interferometer, especially a white light interferometer, is powerful tool for this purpose. An oblique interferometer is also possible to reduce the sensitivity of the fringe^{2,3)}. Abramson proposed an oblique incidence interferometer by using a right-angled prism²⁾ but there is a limitation of incident angle to increase the fringe interval because of a critical angle. To overcome this problem, we propose a super oblique incidence interferometer using a sub-wavelength structure (SWS) prism with the anti-reflection effect. Recently, many papers related a SWS for anti-reflection effect have been proposed⁴⁻⁷⁾. The period of width and height of SWS is almost equal to the wavelength of the incident light. An anti-reflection effect of moth eye structure was first discovered by Bernhard⁴⁾. A vector model for

SWS was first proposed by Moharam called the rigorous coupled wave analysis (RCWA)⁵⁾. It is powerful tool for accurate analysis of the diffraction of electromagnetic waves by periodic structure. We have proposed a model error resulted at the numerical model divided SWS into several sections of model by the RCWA⁷⁾. Most of the anti-reflection studies have been for vertical angle of the incident light. We study optical behaviors at SWS for oblique incidence. Experimental results are compared accurately with simulation results of simulation model by using AFM.

2. Super oblique incidence interferometer by SWS prism

2.1 Experimental setup

Figure 1 is an optical configuration for a super oblique incidence interferometer using SWS prism. This interferometer is based on Mach-Zehnder type with two pair of SWS prism. A beam expander (L1, L2) and two half mirror (HM1, HM2) are used for collimate and tilt of the light. The incident beam is divided into a reference and a object beam by HM1 and HM2. One of the prisms is faced to the sample as an object beam and the other is used as a reference to the M2 mirror. A light source is possible to use both a white light and a monochromatic light. Its sensitivity can be adjusted by changing the incident angle θ to the sample. The height of a sample $h(x,y)$ is shown

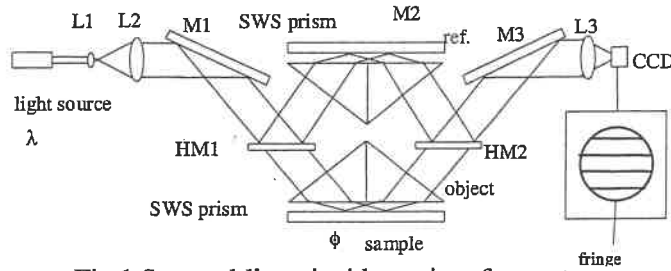


Fig.1 Super oblique incidence interferometer

$$h_{(x,y)} = \frac{\lambda}{4 \cos \theta} \cdot \phi_{(x,y)}, \quad (1)$$

where λ is the wavelength and $\phi(x,y)$ is the phase difference between the reference and the object beam. The intensity $I(x,y)$ of the interference fringe captured by the CCD camera is expressed as,

$$I_{(x,y)} = a^2 + b^2 + 2ab \cos(\phi_{(x,y)} + \delta). \quad (2)$$

Here, a and b are the bias of the reference and the object beam. δ is the phase shift. Four step phase-shifting technique is employed to analyze this fringe.

In case of a white light interferometer, we use the reference SWS prism to compensate wavelength dispersion. However, we can remove it when a monochromatic light source is utilized.

2.2 Fabrication of SWS prism

A SWS prism with anti-reflection effect is illustrated in Fig.2. Figure 2 (a) is a detailed structure of SWS. The requirement of this structure is to fabricate a cycle of width and height equal to the wavelength. An oblique

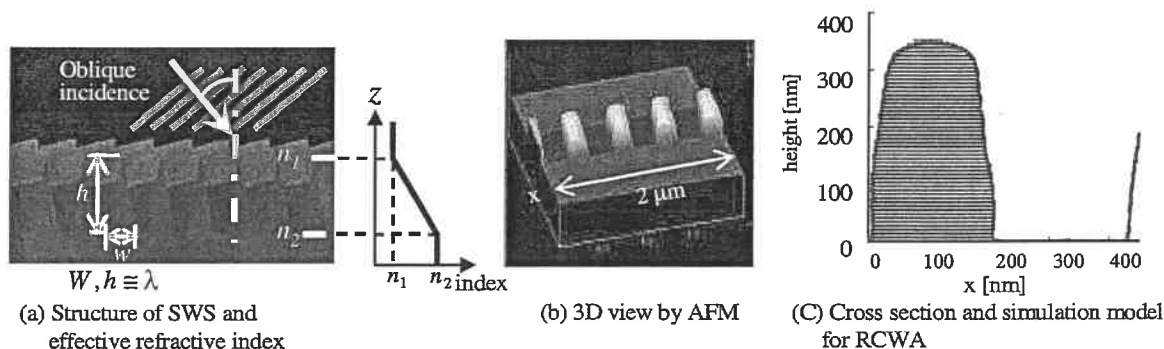


Fig.2 Sub-wavelength structure of anti reflection effect for oblique-incidence

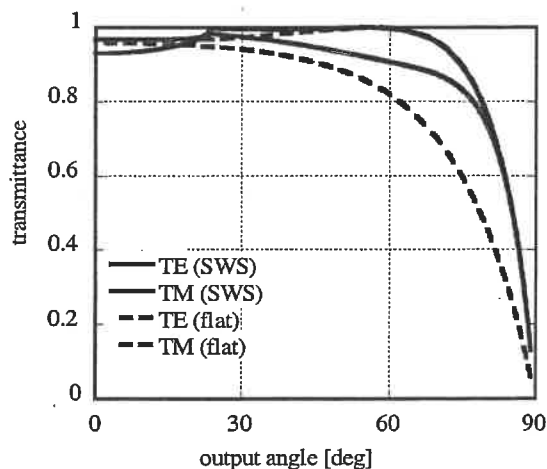
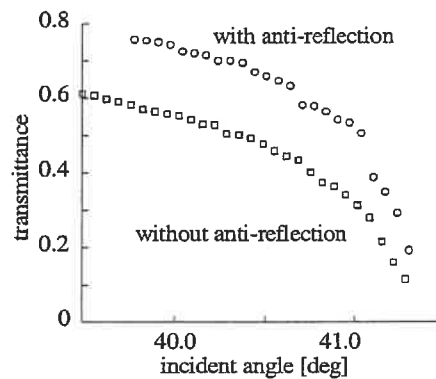
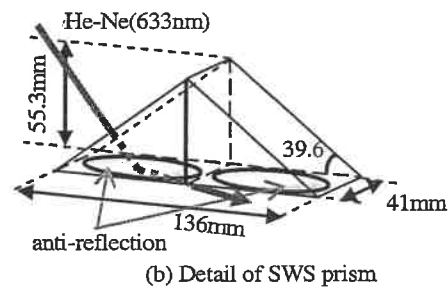
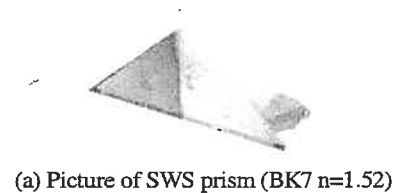


Fig.3 RCWA simulated results of transmittance

incidence wave transmitted from region I (refractive index: n_1) to region II (refractive index: n_2). The refractive indices between two mediums along optical axis are changing slowly. Effective refractive index is obtained on the average of refractive indices of z axis in Figure 2(a). This means the Fresnel reflection at a boundary is also reduced. Figure 2(b) shows a surface profile of the manufactured SWS measured by a critical dimension (CD)-AFM. Its cross-section is shown in Fig.2(c). As this result, a height of SWS is 350 nm, a width is 197 nm and a period is 433 nm.

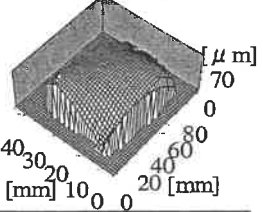
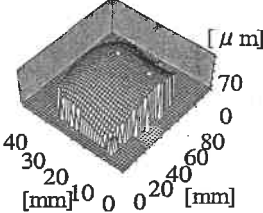
A transmittance of SWS for the oblique incidence has been simulated by using RCWA. The surface profile for this simulation is measured by CD-AFM. The equality horizontal lines in Fig.2(c) mean the simulation



(c) External result of the transmittance

Fig.4 SWS prism

Table 1 Experimental result of surface profile measurement

Incident angle[°]	interferogram	phase map	surface profile
75 ($\Delta h = 1.2\mu m$)			
86 ($\Delta h = 4.5\mu m$)			

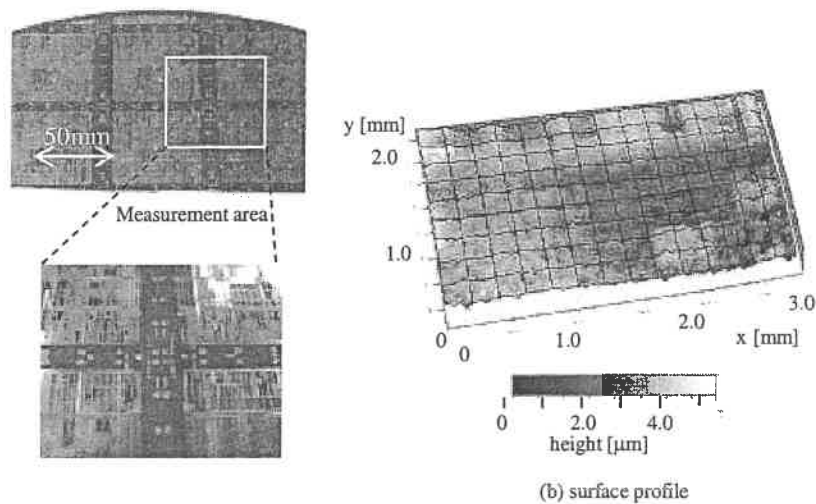


Fig.5 Surface profile of silicon wafer with circuit pattern

model for the SWS and the number of layers is 50⁷⁾. Figure 3 shows the transmittance along output angle compared the SWS prism with the flat surface by RCWA. A wavelength of incident wave is 632.8 nm and polarizations of the incident wave are p and s-polarization. The reflectance of TE mode is lower than flat surface at all incident angles.

Figure 4 slows an experimental result of the anti-reflection effect for the oblique incidence of SWS prism. The SWS prism is made of BK7 glass and polymer film of the sub-wavelength structure shown in Fig.4(a). The

size of the prism is illustrated in Fig.4(b). The two parts of SWS, as shown circled area, glued on the bottom of a prism SWS. Figure 4(c) is the experimental result. The transmittance by SWS prism is much higher than by without SWS. The transmittance of 41° is succeeded to increase more than 50%.

3. Experimental results of surface profile measurement

We measured a piece of silicon wafer by the super oblique incidence interferometer shown in Fig.1. In this experiment, we utilized a He-Ne laser at 632.8nm as a light source. A reference mirror (M2) is moved for phase-shifting by a piezoelectric actuator (PZT, NEC-TOKIN).

Table 1 shows two difference interferogram, phase map and surface profile given by different incident angles. The silicon wafer is taped down on the glass substrate. An oblique incident angles are 75° and 86° . A fringe space of 86° is wider than that of 75° . We can measure a smooth surface profile such as a silicon wafer by this method. The waviness of this silicon is $47\mu\text{m}$.

Figure 5 shows the measured result of a silicon wafer with multilayer interconnection of circuit patterns as Low-k + Cu -DD (Duel Damascene) with CMP (Chemical Mechanical Polishing). Its measurement area is 3×2 mm. We can succeed to measure wafer surface even if the large steps and waviness .

4. Conclusion

We have proposed a super oblique incidence interferometer by SWS prism. We succeeded to increase an efficient transmittance at the oblique incidence by the SWS prism under the condition of the vicinity or critical angle. A shape of the structure is a trapezoid with height and width of sub-wavelength order periodically. The experimental results of transmittance agree well with the calculated results by RCWA. The incident angle of the oblique interferometer could be achieved 86° . The measurement of a silicon wafer with multilayer interconnection of circuit patterns was succeeded to measure by this interferometer.

The authors thank to Japan Veeco for helping the AFM measurement.

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Interferometric Thickness Calibration of 300 mm Silicon Wafers

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Abstract: The "Improved Infrared Interferometer" (IR³) at the National Institute of Standards and Technology (NIST) is a phase-measuring interferometer, operating at a wavelength of 1550 nm, which is being developed for measuring the thickness and thickness variation of low-doped silicon wafers with diameters up to 300 mm. The purpose of the interferometer is to produce calibrated silicon wafers, with a certified measurement uncertainty, which can be used as reference wafers by wafer manufacturers and metrology tool manufacturers. We give an overview of the design of the interferometer and discuss its application to wafer thickness measurements. The conversion of optical thickness, as measured by the interferometer, to the wafer thickness requires knowledge of the refractive index of the material of the wafer. We describe a method for measuring the refractive index which is then used to establish absolute thickness and thickness variation maps for the wafer.

Keywords: 300 mm silicon wafers, wafer thickness, wafer thickness variation (TTV), Interferometry, Infrared interferometer

INTRODUCTION

The International Technology Roadmap for Semiconductors (ITRS) [1] projects an exposure site flatness requirement of ≤ 51 nm and a nanotopography requirement of ≤ 13 nm by 2010. Nanotopography is the component of flatness error with spatial frequencies smaller than 0.05mm^{-1} . These stringent requirements for wafer flatness at the exposure site are imposed by the physics of the photolithography process. For diffraction limited exposure objectives, the smallest achievable linewidth, L , is determined by the exposure wavelength and numerical aperture, NA , of the objective:

$$L = k_1 \frac{\lambda}{NA} \quad (1)$$

k_1 is a lithography process dependent factor which depends, for example, on properties of the resist. Creating smaller integrated circuit features compels moving to shorter wavelengths and larger numerical apertures. When the depth of focus,

$$D = k_2 \frac{\lambda}{NA^2} \quad (2)$$

is reduced, this - in turn - limits the allowable flatness variation at the exposure site. Again, k_2 is a process dependent factor. The tight limits on flatness and nanotopography create new challenges at two stages of the integrated circuit (IC) manufacturing process. Wafer manufacturers must improve the polishing processes

to meet the requirements for wafer flatness and nanotopography. Manufactures of wafer testing equipment need tools which allow measurement of flatness and nanotopography with sufficient accuracy and spatial resolution. Optical tools are beginning to replace wafer flatness metrology tools based on capacitance gages or airflow gages. Our research addresses the needs of both wafer manufacturers and wafer testing equipment manufacturers for 300 mm silicon reference wafers with a calibrated thickness and thickness variation, which are measured with low uncertainty, and can be used for calibration and validation purposes.

In the following sections we will describe the "Improved Infrared Interferometer" (IR³) of the National Institute of Standards and Technology (NIST). IR³ was developed to measure the thickness and thickness variation of low-doped silicon wafers with diameters up to 300 mm.

THE IR³ INTERFEROMETER

Undoped silicon is transparent to light at wavelengths larger than approximately 1100 nm. The thickness and thickness variation of wafers made from silicon, or other infrared optical materials, can be characterized using the well established methods of optical interferometry, which achieve very low measurement uncertainties. NIST's IR³ interferometer is a multi-configuration interferometer that can be used as a Twyman-Green interferometer, a

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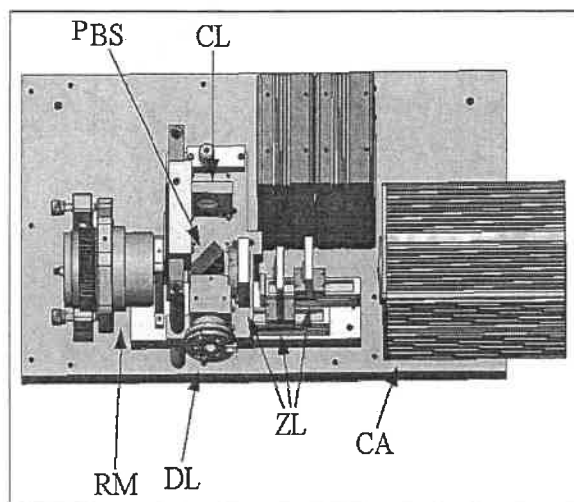


Figure 1. NIST Improved Infrared Interferometer (IR³). The main components of the interferometer are: collimator lens (CL), polarizing beam splitter (PBS), phase-shifting reference mirror mount (RM) for the Twyman-Green configuration, diverger lens (DL), zoom lens system (ZL), and camera (CA). The footprint of the interferometer is 250 mm x 410 mm.

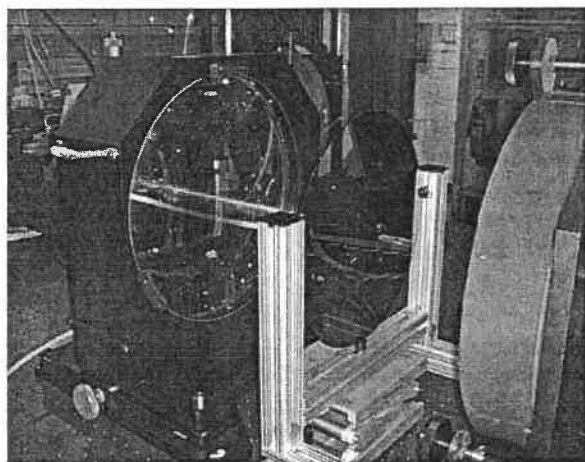


Figure 2: Test end of the IR³ setup for thickness variation measurements of 300 mm wafers. Shown are the collimator lens together with a silicon wafer and a return flat.

Fizeau interferometer, or a Haidinger interferometer. Precursors to the current instrument have been described by Parks *et al.* [2] and Schmitz *et al.* [3]. Figure 1 shows a solid model of the interferometer. Light from a tunable external cavity diode laser is delivered to the interferometer through a polarization-maintaining fiber. The interferometer operates at a wavelength of 1550 nm. At this wavelength,

which is within the C-band of fiber-optics communication, laser sources are readily available. The plane of polarization is inclined by 45° with respect to the plane of the interferometer. After collimation by a lens (CL) a polarizing beam splitter (PBS) creates two beams with orthogonal polarization, one traveling to the diverger lens (DL), the other to the reference mirror (RM) for the Twyman-Green configuration. Both beams are circularly polarized by precision $\lambda/4$ -plates. For measurements of 300 mm wafers, the test beam must be expanded. This is accomplished with an $f/3$ diverger objective (DL) and the large collimator lens shown in Figure 2. A zoom lens system (ZL) images the wafer under test on the camera (CA). Commercial phase-shifting software is used to make phase measurements either by shifting the wavelength of the laser or by electro-mechanical shifting of the reference mount. The polarization optics of the interferometer prevent unwanted coherent reflections from reaching the camera and thus eliminates "ghost fringes".

MEASUREMENT SETUPS

IR³ is a flexible tool, which is configurable in several ways. Figure 3 indicates two of its configurations, the Fizeau (A) and Twyman-Green (B) configurations. Either can be used for measuring the thickness variation of wafers.

In the Fizeau configuration the interferometer test beam is expanded to 300 mm by the collimator lens. The wafer itself becomes the Fizeau cavity of the interferometer. Reflected light from both wafer surfaces returns to the interferometer's camera where interferograms are measured. The laser wavelength is varied to implement phase shifting interferometry, and the resulting height maps directly represent the optical thickness variation of the test wafer. To obtain the physical thickness variation of the wafer this must be divided by the refractive index of the wafer material. The Fizeau configuration has many advantages for thickness variation measurements. The interferometer cavity is a solid and is unaffected by vibration and turbulence is absent. In addition, the coherence length of the laser source can be reduced because the distance between the wafer surfaces is small. This completely eliminates coherent reflections from optical

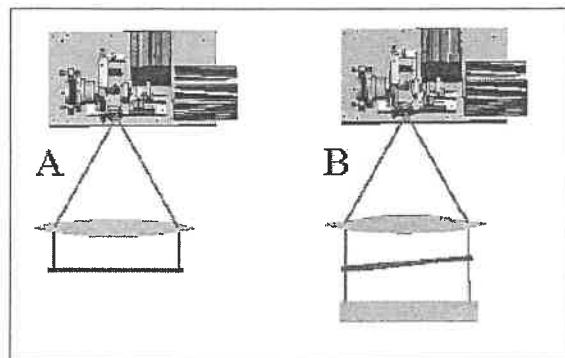


Figure 3. Two configurations of the IR3 interferometer for wafer thickness variation measurements. A is a Fizeau configuration with the wafer as interferometer cavity, B is a Twyman-Green configuration.

surfaces in the interferometer which otherwise lead to measurement errors.

Sometimes measurements cannot be made in Fizeau mode, for example when a wafer is not polished on both sides or if the free spectral range of the wafer cavity exceeds the tuning range of the laser source. The Twyman-Green setup B, shown in Figure 3, must then be used. The wafer is tilted slightly to prevent the light reflected by the wafer surfaces from returning to the interferometer. Instead, the test beam, after having passed through the wafer, is returned to the interferometer with a return flat after passing through the wafer a second time. Phase shifting is implemented by mechanically moving the Twyman-Green reference mirror with a piezo-electric phase shifter. Now two measurements are required, one with the wafer in the interferometer cavity and one without. The optical thickness variation Θ of the wafer is then calculated as follows:

$$\Theta = \frac{1}{2} \frac{O_e - O_w}{n_{Si} - n_{air}}, \quad (3)$$

where O_e is the optical path difference for the empty interferometer and O_w is the optical path difference with the wafer inserted. As in the case of the Fizeau configuration, the refractive index n_{Si} must be known to determine the thickness variation. n_{air} is the refractive index of air which is about 1.00027 for normal laboratory conditions at 1550 nm.

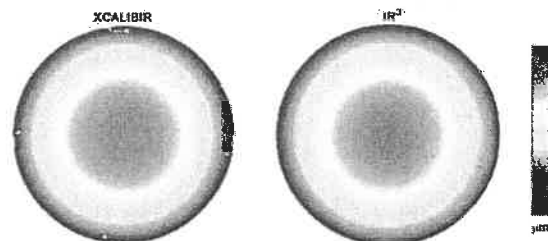


Figure 4. Thickness variation of a 300 mm diameter, 2 mm thick fused silica wafer measured with NIST's XCALIBIR interferometer at 633 nm (left) and with the IR³ interferometer at 1550 nm (right).

TEST MEASUREMENTS

To validate the performance of the IR³ interferometer we measured the thickness variation of a 300 mm diameter, 2 mm thick fused silica wafer with IR³ and compared the results with thickness variation measurements made with NIST's XCALIBIR interferometer which is a 300 mm aperture, phase-shifting Fizeau interferometer working at a wavelength of 633 nm. Fused Silica is a stable and well-characterized material that is transparent at 633 nm and at 1550 nm. For the XCALIBIR measurement, a Fizeau cavity was set up with two 300 mm diameter reference flats and the empty cavity was measured. Then the silica wafer was inserted into the cavity and another measurement was made. The thickness variation was calculated from these two measurements in the manner described in the previous section and Equation 3. In IR³ the same silica wafer was measured in Fizeau configuration with wavelength phase-shifting. For the determination of the physical thickness variation of the wafer the measurement of the refractive index of fused silica by Malitson [4] was used. Both measurements of the thickness variation of the fused silica wafer are shown in Figure 4 side by side. A careful comparison of the measurements shows essentially no difference between the two.

REFRACTIVE INDEX

When an interferometer is used to measure thickness and thickness variation of a wafer, an additional measurement of the refractive index of the wafer material must be made to determine the physical thickness of the wafer. In this section, we describe a method for measuring the refractive index of the wafer material using the

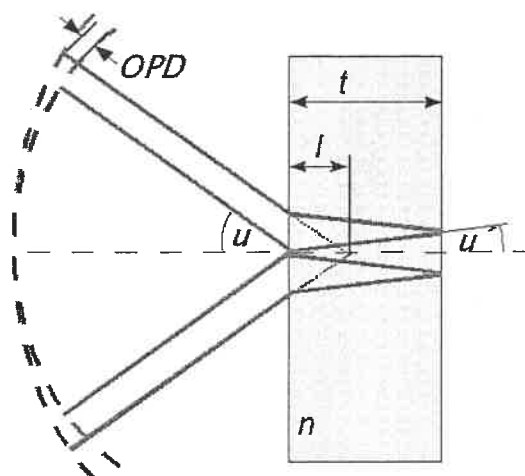


Figure 5. Reflection of the converging light cone with half cone angle u from the diverger objective by a wafer near the focus.

IR³ interferometer. Parks *et al.* [2] have described a Haidinger interferometer for thickness variation measurements of wafers in a diverging beam. We use a variant of this method to measure the refractive index of the wafer material at one point of the wafer. The wafer is placed into the interferometer at the focus of the diverger lens as shown in Figure 5. Figure 6 shows the fringes, and the resulting height map, that are seen when a silicon wafer is placed in the interferometer. No additional optics is needed for the measurement. Light from the wafer front surface is reflected back into the interferometer. This is the reference beam. The beam passing through the wafer is reflected back into the interferometer by the wafer's back surface. The optical path difference (OPD) between the two wavefronts at the margin of the light cone is given by [5]:

$$OPD = \frac{1}{2} l \sin^2 u, \quad (4)$$

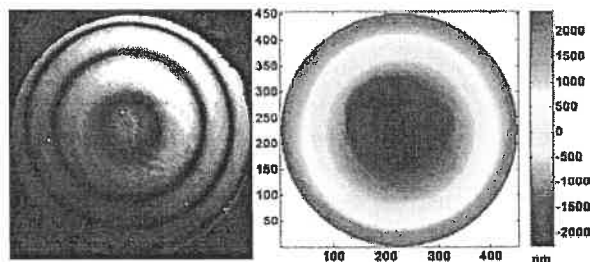


Figure 6. Interferogram and height map for a wafer inserted near the focus of the diverger objective.

where l is the focus separation of the reference and test light cones, which is given by:

$$l = \frac{2t \tan u'}{\tan u}. \quad (5)$$

u is the half angle of the light cone created by the diverger objective and u' is the half angle of the refracted light cone inside the wafer. These two angles are related by Snell's Law:

$$\sin u' = \frac{\sin u}{n_{si}}. \quad (6)$$

It follows from Equations 4 to 6 that the refractive index of the wafer material can be calculated once the OPD for the marginal ray, angle u , and wafer thickness t are known. To confirm this method yields accurate results for the refractive index, we have measured the refractive index of the fused silica wafer mentioned in the previous section because the refractive index of fused silica is well known ([4]). For the measurement of the half angle of the light cone made by the diverger objective an f/3 return sphere with a radius of about 1m was set up confocal with the diverger. The return sphere was then moved a small distance, l , and the resulting defocus was measured with the interferometer. A light cone half angle of $8.115(6)^\circ$ was calculated using Equation 4¹. Next, the fused silica wafer was placed in the interferometer near the focus, the resulting height map was measured with the interferometer, and the Zernike coefficient a_{20} (focus shift, or power) for this map was calculated. Its value was $28.283(1) \mu\text{m}$. The physical thickness of the wafer at the same location was determined with a mechanical contact measurement and found to be $2062(1) \mu\text{m}$. Using Equations 4 to 6 one can calculate the refractive index of fused silica, which is $1.442(3)$ at 1550 nm . This is in good agreement with Malitson's value of 1.444 [4].

WAFER THICKNESS VARIATION

The thickness calibration of a silicon wafer is a two-step process. First, the refractive index of the wafer material is measured at one (or more) locations of the wafer with the procedure outlined above. For these measurements the

¹ Combined standard uncertainties in the least significant digit of a value are stated in brackets.

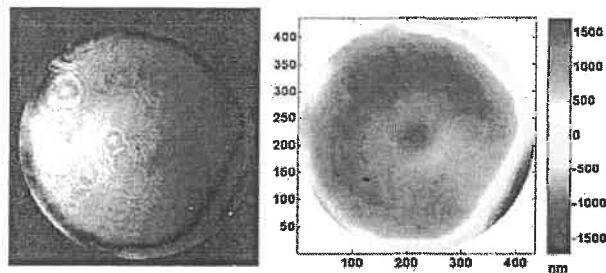


Figure 7. Interferogram and optical thickness variation of a 300mm diameter silicon wafer. The height unit is nm and the x and y coordinates are pixel numbers.

silicon wafer was placed in the interferometer so that the diverger objective focus illuminates a small area about 10 mm from the wafer notch in the radial direction. Figure 6 shows the interference fringes and the resulting height map. The OPD of the meridional ray was determined by measuring the "focus shift" Zernike term of the height map in Figure 6 with phase-shifting interferometry. The thickness at the illuminated point of the wafer was measured with an electro-mechanical indicator and it is 770(1) μm . Using Equations 4 to 6 it follows the refractive index of the wafer material is 3.471(8) at 1550 nm. The refractive index measurement was repeated at the center of the wafer to detect any radial changes in the refractive index. The refractive index at the wafer center is 3.492(8). These values are in good agreement with those reported by Primak [6] and Villa [7]. We observed a statistically significant difference between the index at the center and at the periphery of the wafer which warrants further investigation. The optical thickness variation of the wafer is depicted in Figure 7, and Figure 8 represents the physical thickness variation of this low-doped 300 mm double-side polished silicon wafer. The repeatability of the measurements is 5 nm peak-to-valley. The physical thickness variation of the wafer is within 1000 nm peak-to-valley. It is clear, this variation in physical thickness cannot be due to a change in refractive index of the magnitude we measured. A reduction of the uncertainty of the refractive index measurement will be necessary before we can determine with certitude which fraction of the optical thickness variation is due to index variation and which is caused by physical thickness variation.

CONCLUSION

Measurements with the "Improved Infrared Interferometer" (IR³) of the National Institute of

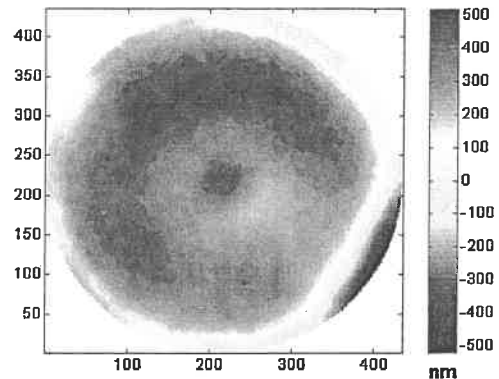


Figure 8. Physical thickness variation map of a 300mm diameter silicon wafer. The height unit is nm and the x and y coordinates are pixel numbers.

Standards and Technology (NIST) show that the interferometer is a promising tool for the thickness characterization of 300 mm silicon wafers with total thickness variation below 1 μm .

ACKNOWLEDGMENTS

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MODELLING OF OPTICAL IMAGE TRANSFORMATIONS BY MEANS OF 4x4 MATRICES

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Keywords: homogenous matrix, matrix algebra, optical reflection and refraction, ray tracing, interferometric metrology modelling.

1 Introduction

High-accuracy positioning of industrial manipulators is usually accomplished by means of laser-interferometric measurement systems. In such systems, beams of coherent light are subject to refractions and reflections in prisms and straightedges attached to various stage subassemblies, both moving and stationary. While analysing such a metrology system, one usually aims at determining the effect of both the stage motion and deviations from the layout geometry on the indications of the interferometers. Many ray tracing tools exist which perform this task numerically, calculating the beam position and orientation at its intersection with each reflection and refraction plane for a predefined configuration of prism and beam geometry. The problem with this approach is that it often requires a laborious layout definition, while the results it produces are purely numerical and thus do not offer a deeper insight into the effects involved.

Modelling techniques for rigid body transformations in 3D space are well known and utilise either 4x4 homogenous transformations or quaternions. This paper argues that certain 4x4 matrices can also represent image transformations associated with optical reflections and refractions on flat planes in 3D space. By representing prisms as matrices, the new method allows analysing their sequences without the need for ray tracing. It also presents an algebraic connection between those image transformation matrices and the motion applied to the reflecting and refracting planes, as well as prisms. The Authors are not aware of any references to 4x4 matrix formulation for purposes of optical analysis and claim that such formulation is innovative in this context.

The new technique is applicable to flat reflection/refraction surfaces. In addition, extracting the optical path in glass becomes possible as an algebraic operation. The new method is particularly useful when employed for symbolic computation in packages such as Mathematica and Maple. The proofs of the postulated image transformations require only elementary algebra.

2 Optical Image

In the context of this document, the optical image of a point under certain transformation will be the limit of the virtual origin of an infinitesimally narrow bundle of rays having undergone the transformation in question. The optical image of a vector, line or plane shall be constructed by aggregation of images of their member points.

2.1 Reflection

The optical image of a point under reflection is positioned on the other side of the reflection plane, at the same distance to it as the original, the segment connecting the image and the original being orthogonal to the reflection plane. It is postulated the a reflection plane satisfying the equation: $ax + by + cx + d = 0$ introduces an image transformation, which can be expressed as a 4x4 matrix:

$$\mathbf{K}_{refl} = \frac{1}{a^2 + b^2 + c^2} \begin{pmatrix} -a^2 + b^2 + c^2 & -2ab & -2ac & -2ad \\ -2ab & a^2 - b^2 + c^2 & -2bc & -2bd \\ -2ac & -2bc & a^2 + b^2 - c^2 & -2cd \\ 0 & 0 & 0 & a^2 + b^2 + c^2 \end{pmatrix} \quad (1)$$

When multiplied by vector or point coordinates associated with the beam origin or direction, the 4x4 transformation yields their optical images in refraction or reflection. By the convention common in the traditional 4x4 spatial transformations, a unity is appended to point coordinates and a zero to vector coordinates to form a column vector of the required size.

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \mathbf{K}_{refl} \cdot \begin{pmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} v'_x \\ v'_y \\ v'_z \\ 0 \end{pmatrix} = \mathbf{K}_{refl} \cdot \begin{pmatrix} v_x \\ v_y \\ v_z \\ 0 \end{pmatrix} \quad (2)$$

Please observe that the reflection transformation is its own inverse, what might be expected (reflecting twice leaves the original unchanged). Furthermore, the determinant of the reflection image transformation matrix is equal to -1 , which represents the change of (left/right-) handedness:

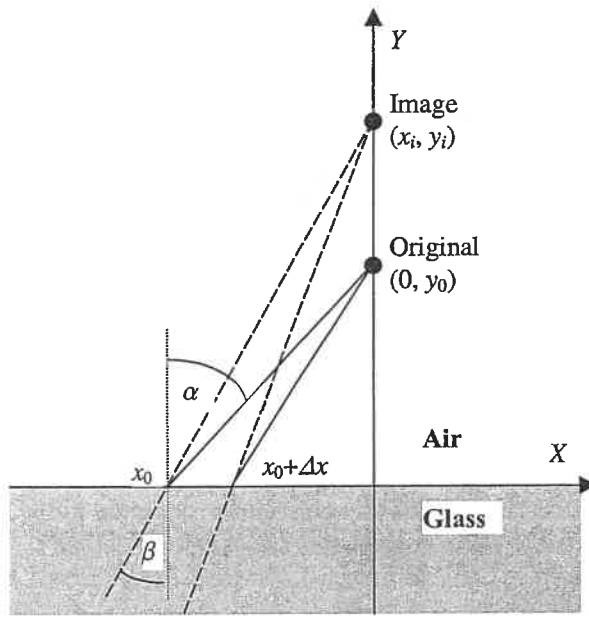
$$\begin{aligned} \mathbf{K}_{refl}^{-1} &= \mathbf{K}_{refl} \\ \det(\mathbf{K}_{refl}) &= -1 \end{aligned} \quad (3)$$

To prove the correctness of the transformation, it is sufficient to show the proper distance equivalence and orthogonality for any plane $\mathbf{s} = (a, b, c, d)^T$ and point $\mathbf{p} = (x, y, z, 1)^T$, taking only 3 coordinates where a vector product necessitates it (we omit the complete derivation for brevity):

$$\begin{aligned} \mathbf{s} \cdot \mathbf{p}' + \mathbf{s} \cdot \mathbf{p} &= \mathbf{s} \cdot (\mathbf{K}(\mathbf{s}) \cdot \mathbf{p} + \mathbf{p}) = \mathbf{s} \cdot (\mathbf{K}(\mathbf{s}) + \mathbf{I}) \cdot \mathbf{p} = \mathbf{0} \cdot \mathbf{p} = 0 \\ (\mathbf{p}' - \mathbf{p})^{[3]} \times \mathbf{s}^{[3]} &= (\mathbf{K}(\mathbf{s}) \cdot \mathbf{p} - \mathbf{p})^{[3]} \times \mathbf{s}^{[3]} = \mathbf{0} \end{aligned} \quad (4)$$

2.2 Refraction

We put forward a hypothesis that in refraction, the optical image is located on the same side of the refraction plane, at a distance n times the original distance, n being the relative index of refraction. As before, the segment connecting the image and the original is orthogonal to the refraction plane. This is an approximation for small incidence angles, equivalent to assuming $\sin(x) \approx \tan(x)$ in the Snellius law of refraction. In general, the image location would depend on the incidence angle. The crucial observation is that up to the linear terms of the incidence angle, the image is angle-invariant, just like in a lens.



It can be shown (cf Figure 1) that for two rays originating from the same point $(0, y_0)$ and refracting at $(x_0, 0)$ and $(x_0 + \Delta x, 0)$ the intersection of their extensions has a limit as their separation Δx approaches 0, which is given by:

$$\begin{aligned} x_i &= -x_0 \left(1 - \frac{1}{n^2} \right) \tan^2 \alpha \approx 0 \\ y_i &= ny_0 \left[1 + \left(1 - \frac{1}{n^2} \right) \tan^2 \alpha \right]^{\frac{3}{2}} \approx ny_0 \end{aligned} \quad (5)$$

By analogy to the reflection image transformation, we postulate that a refraction plane satisfying the equation: $ax + by + cx + d = 0$, introduces the following image transformation:

Figure 1 The construction of an image in refraction.

$$\mathbf{K}_{refr} = \frac{1}{a^2 + b^2 + c^2} \begin{pmatrix} na^2 + b^2 + c^2 & (n-1)ab & (n-1)ac & (n-1)ad \\ (n-1)ab & a^2 + nb^2 + c^2 & (n-1)bc & (n-1)bd \\ (n-1)ac & (n-1)bc & a^2 + b^2 + nc^2 & (n-1)cd \\ 0 & 0 & 0 & a^2 + b^2 + c^2 \end{pmatrix} \quad (6)$$

$\mathbf{K}_{refr}$ can be applied in a manner analogical to $\mathbf{K}_{refl}$ in (2). Please also observe that $\mathbf{K}_{refl}$ is a special case of $\mathbf{K}_{refr}$, when $n = -1$. Furthermore, inverting the transformation matrix is equivalent to inverting the refraction index, as changing the medium from air to glass and back leaves the original unaltered, while the determinant predictably indicates a stretch in one direction:

$$\begin{aligned} \mathbf{K}_{refl}^{-1}(n) &= \mathbf{K}_{refl} \left(\frac{1}{n} \right) \\ \det(\mathbf{K}_{refr}) &= n \end{aligned} \quad (7)$$

As before, to prove the correctness of the transformation, it is sufficient to show the proper $n:1$ distance relation and orthogonality for any plane $\mathbf{s} = (a, b, c, d)^T$ and point $\mathbf{p} = (x, y, z, 1)^T$, taking only 3 coordinates where a vector product necessitates it (we omit the full derivation for brevity):

$$\begin{aligned} \mathbf{s} \cdot \mathbf{p}' - n \cdot \mathbf{s} \cdot \mathbf{p} &= \mathbf{s} \cdot (\mathbf{K}(\mathbf{s}) \cdot \mathbf{p} - n \cdot \mathbf{p}) = \mathbf{s} \cdot (\mathbf{K}(\mathbf{s}) - n \cdot \mathbf{I}) \cdot \mathbf{p} = \mathbf{0} \cdot \mathbf{p} = 0 \\ (\mathbf{p}' - \mathbf{p})^{[3]} \times \mathbf{s}^{[3]} &= (\mathbf{K}(\mathbf{s}) \cdot \mathbf{p} - \mathbf{p})^{[3]} \times \mathbf{s}^{[3]} = \mathbf{0} \end{aligned} \quad (8)$$

2.3 Transformation of a Plane

In order to obtain the image $\mathbf{s}' = (a', b', c', d')^T$ of a plane $\mathbf{s} = (a, b, c, d)^T$ under reflection or refraction $\mathbf{K}$, the image transformation matrix has to be *inverted and transposed*:

$$\mathbf{s}' = (\mathbf{K}^{-1})^T \mathbf{s} \quad (9)$$

To prove it, we first observe that if a point $\mathbf{p} = (x, y, z, 1)^T$ satisfies the equation of the original plane, the scalar product $\mathbf{s}^T \cdot \mathbf{p} = 0$. The image $\mathbf{p}' = (x', y', z', 1)^T$ of point $\mathbf{p}$ under $\mathbf{K}$ is given as $\mathbf{p}' = \mathbf{K} \cdot \mathbf{p}$ and it must satisfy the equation of the transformed plane $(\mathbf{s}')^T \cdot \mathbf{p}' = 0$:

$$(\mathbf{s}')^T \mathbf{p}' = \left[(\mathbf{K}^{-1})^T \mathbf{s} \right]^T \mathbf{K} \cdot \mathbf{p} = \mathbf{s}^T \mathbf{K}^{-1} \mathbf{K} \cdot \mathbf{p} = \mathbf{s}^T \mathbf{p} = 0 \quad (10)$$

2.4 Prism and Optical Path Transformation

A prism is a sequence of planes; it therefore possesses an image transformation being the product of image transformation matrices associated with the individual refraction and reflection planes. Due to the chosen left-associativity of the $\mathbf{K}$ operator, the order of matrices is the reverse temporal order in which the beam encounters the planes on its way:

$$\mathbf{K}_{prism} = \mathbf{K}_{refr} \left(\mathbf{s}_N, \frac{1}{n} \right) \cdot \mathbf{K}_{refl} (\mathbf{s}_{N-1}) \cdot \dots \cdot \mathbf{K}_{refl} (\mathbf{s}_2) \cdot \mathbf{K}_{refr} (\mathbf{s}_1, n) \quad (11)$$

The first refraction plane (from air to glass) must be taken with the relative index of refraction n , the last refraction plane (from glass to air) with the relative index $1/n$. Naturally, the transformation matrices for all prisms participating in a given optical path can be multiplied by each other to produce a path transformation:

$$\mathbf{K}_{path} = \mathbf{K}_M \cdot \mathbf{K}_{M-1} \cdot \dots \cdot \mathbf{K}_2 \cdot \mathbf{K}_1 \quad (12)$$

We can utilise the path transformation matrix to obtain the image of, for instance, an interferometric beam source or target (understood as point, vector or plane). This will allow analysing beam walking, angular deviations and other properties, both symbolically and numerically.

3 Motion Effect

3.1 Prism Motion

To determine the image transformation matrix of a prism to which a 4x4 spatial transformation $\mathbf{T}$ has been applied, one would normally apply $\mathbf{T}$ to all the reflection and refraction planes, construct a new chain of $\mathbf{K}$ matrices and multiply them out. (Applying a spatial transformation to a plane obeys the same rule as in (9), since the proof was not restricted to optical transformations.)

$$\mathbf{K} = \mathbf{K} \left((\mathbf{T}^{-1})^T \mathbf{s}_N \right) \cdot \mathbf{K} \left((\mathbf{T}^{-1})^T \mathbf{s}_{N-1} \right) \cdot \dots \cdot \mathbf{K} \left((\mathbf{T}^{-1})^T \mathbf{s}_1 \right) \quad (13)$$

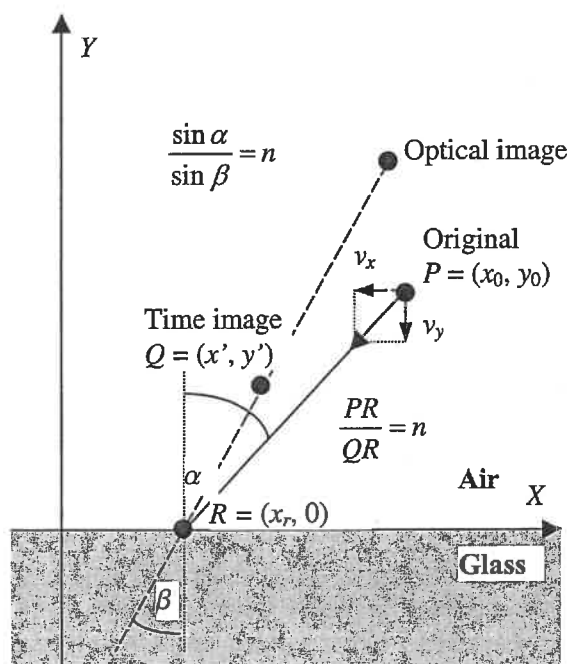
However, this is a very laborious process and it fails quickly in symbolic computation. We will show that, for a spatially transformed prism, $\mathbf{K}$ is a direct function of $\mathbf{T}$. It cannot be a function of the prism's individual orientation angles R_x, R_y, R_z , since those angles depend on the orientation convention (e.g. ZYX, XYZ, etc.), while the result cannot exhibit any such dependency. It is possible to prove that $f(\mathbf{T})$ must have the following algebraic form:

$$\mathbf{K} = f(\mathbf{T}) = \mathbf{T} \cdot \mathbf{K}_0 \cdot \mathbf{T}^{-1} \quad (14)$$

(The proof is based on the observation that applying an additional spatial transformation $\Delta\mathbf{T}$ to the prism and to the original must result in a similarly rotated image, as it is equivalent to transforming our frame of reference.) This result shows that every prism, irrespective of its number of surfaces, can be associated with a constant image transformation $\mathbf{K}_0$, which is derived at its nominal position and orientation. The significance of this fact is that $\mathbf{K}_0$ usually has fewer independent variables than the number of surfaces would indicate. Moreover, both the motion and the surface misalignment become apparent in a chain of matrix multiplications and are thus amenable to standard analytical methods, such as differentiation.

4 Time-Image

One should realise that the optical image transformation in refraction *contracts* the optical path in glass, instead of *expanding* it, which is what one would normally expect. For that reason, while perfectly suitable for analysing beam walking, it is rather inconvenient for deriving optical path lengths. In this section we present an alternative refraction transformation, one which straightens out the optical path while correctly accounting for the changes of the optical medium in an automatic and transparent manner.



We introduce the *time-image* of a point under refraction as the virtual source of rays having undergone the refraction and satisfying the condition that **the time the ray travelled from the original to the refraction point is equal to the time it would have travelled from the image to the refraction point if it was immersed in the new medium**. In this manner, the optical path length from the original in two media is equal to the optical path length from the image in the new medium. Such a definition presupposes a given direction of the image-forming beam. Please observe that for an air-to-glass refraction, the traditional optical image is roughly n times further away from the glass surface than the original, while the time-image is roughly n times closer, where n is the refraction index.

It can be shown (cf Figure 2) that the time-image of an original point (x_0, y_0) , projected by a beam arriving at an incidence angle α is given by the following equations:

Figure 2 The construction of a time-image in refraction.

$$\begin{aligned} x' &= x_0 - y_0 \left(1 - \frac{1}{n^2}\right) \tan \alpha \\ y' &= \frac{y_0}{n} \sqrt{1 + \left(1 - \frac{1}{n^2}\right) \tan^2 \alpha} \end{aligned} \quad (15)$$

To determine the time-image transformation of an arbitrary original point $\mathbf{p} = (x_0, y_0, z_0)$ in a 3-dimensional space, for an arbitrary refraction plane given by the equation $ax + by + cz + d = 0$, and an arbitrary beam direction $\mathbf{v} = (v_x, v_y, v_z)$, we first have to transform spatially the original, the plane and the beam to the nominal arrangement (Figure 2), apply the time-image operation and transform the image back to chosen arrangement. For brevity, we present here only the final result:

$$\mathbf{K} = \mathbf{I} + \left[\frac{\left(\gamma - \frac{1}{n^2}\right)}{(a^2 + b^2 + c^2)} \begin{pmatrix} a \\ b \\ c \\ 0 \end{pmatrix} - \frac{\left(1 - \frac{1}{n^2}\right)}{(av_x + bv_y + cv_z)} \begin{pmatrix} v_x \\ v_y \\ v_z \\ 0 \end{pmatrix} \right] \cdot (a \quad b \quad c \quad d) \quad (16)$$

where:

$$\gamma = \frac{1}{n} \sqrt{1 + \left(1 - \frac{1}{n^2}\right) \tan^2 \alpha} \quad (17)$$

All the significant relations which we derived for the optical image, such as (2), (9) and (11), remain valid for the time-image as well. However, now the transformation matrix does depend on the incidence angle α and one has to transform the beam direction as well. The most practical approach is to operate on beam wavefront planes by setting $v_x = a$, $v_y = b$ and $v_z = c$ in (16), as the beam direction is perpendicular to the wavefront by definition.

5 Conclusions

Our paper shows that optical refraction and reflection on flat planes can be represented by means of a class of 4x4 matrices. When multiplied by vector or point coordinates associated with the beam origin, direction or wavefront plane, the 4x4 transformation yields their images in refraction or reflection. In this manner, the successive refractions and reflections the beam encounters on its way can be represented as a matrix product, while characteristic 4x4 matrices can be attributed to specific prism designs. Significantly, there exists a relationship between the optical 4x4 transformations and the homogenous 4x4 transformations, which are commonly used for modelling of rigid body motions in 3D space. This relationship allows for a seamless algebraic treatment of motion and optical phenomena in stage interferometry.

Furthermore, the time-image transformation we have introduced automatically accounts for the different media along the optical path length. This approach obviates a conscious consideration of the intersection between the beam and the refraction plane, thus offering "ray tracing without tracing rays". Finally, the 4x4 optical transformations allow a degree of symbolic analysis which could not be afforded with traditional ray tracing. This, in turn, leads to explicit model expressions for the indicated optical path lengths which are functions of motion and geometry errors. Such expressions can be useful as coordinate transformations required for motion control and as parametric models in calibration of the position metrology.

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